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What to Do about Canada's Declining Math Scores

With mounting evidence showing the shortcomings of discovery-based instruction, it's time to put more emphasis on direct instruction in math to help reverse the decline in student scores.

Anna Stokke

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The Study In Brief

The declining performance of Canadian students on international math assessments should worry Canadians and their provincial governments. Strong mathematics knowledge is required for success in the workforce, and early achievement in math is one of the best predictors of later academic success and future career options.

Between 2003 and 2012, all but two Canadian provinces showed statistically significant declines in math scores on international exams administered by the Organization for Economic Cooperation and Development. In several provinces, the percentage of students performing at the lowest levels in math significantly increased while the percentage of students performing at the highest levels significantly decreased, suggesting that more students are struggling and fewer students are excelling in math. It should be a policy priority to halt these trends and to improve math achievement for Canadian children.

In this *Commentary*, I examine domestic and international evidence regarding three areas of provincial education programs that could play an important role in halting the downward trend in math scores. I make three main recommendations regarding best teaching practices in math, the math curriculum, and the math knowledge of future teachers.

Best teaching practices in math have been at the forefront of discussions regarding declining math scores in Canada. Discovery-based instruction – also called problem-based, inquiry, experiential, and constructivist learning – has become popular in North America in recent years, pushing aside direct instruction techniques, like times table memorization, explicit teacher instruction, pencil-and-paper practice, and mastery of standard mathematical procedures.

Based on international and domestic evidence, this *Commentary* finds that studies consistently show direct instruction is much more effective than discovery-based instruction, which leads to straightforward recommendations on how to tilt the balance toward best instructional techniques.

Student fluency with particular math concepts, such as fraction arithmetic, in early and middle years has been shown to predict future math success. This *Commentary* recommends that provincial math curricula be rewritten to remove ineffective pedagogical directives and to stress specific topics, at appropriate grade levels, that are known to lead to later success in math.

Evidence shows that teachers who are most comfortable and knowledgeable with the content they are required to teach tend to transmit that knowledge best to students. This *Commentary* suggests that future early and middle-years teachers be required to pass a math-content licensure exam prior to receiving certification to teach mathematics.

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Canadian students need a strong foundation in mathematics to succeed in school and to contribute to society as numerate citizens.

Some knowledge of mathematics is required for most careers, including business, nursing, construction and various trades, while a high level of mathematics knowledge is required for fields such as medicine, engineering, economics and careers in science.

Early achievement in mathematics is a strong predictor – even more so than reading skills – of later academic achievement, financial success, and future career options (Charette and Meng 1998; Duncan et al. 2007; Duncan 2011; Romano et al. 2010). Students who struggle early in math struggle later on: sixth graders who fail math have less than a one-in-five chance of starting twelfth grade on time, and only 19 percent graduate on time or within a year (Balfanz 2007).

It is thus important for the future success of Canada's children and economy that provinces take significant steps to halt the decline in Canadian students' math scores on international exams.

A number of factors generally work together to contribute to educational success. In this *Commentary*, I examine domestic and international evidence regarding three major areas of provincial education programs that might be contributing to the downward trend in math scores or that could play an important role in halting this trend. I make the following three main recommendations, which involve teaching practices in mathematics, the math curriculum and the math content knowledge of future teachers:

- As a rule of thumb, teachers should be encouraged to follow an 80/20 rule, favouring direct instructional techniques over discovery-based instructional techniques.
- Provincial math curricula should be rewritten to remove ineffective pedagogical directives, and streamlined to emphasize specific concepts, at appropriate grade levels, that have been shown to lead to later success in mathematics.
- Future early and middle-years math teachers should be required to complete a minimum of six credit hours of math content courses while in university, and to pass a math content licensure exam in which they demonstrate fluency in topics from the kindergarten to eighth grade math curriculum before they can obtain certification to teach mathematics.

The Decline in Canadian Math Skills

The declining performance of Canadian students on international math assessments should be of concern to Canadians and their provincial governments, under whose jurisdiction education falls. Sixty-five jurisdictions took part in the 2012 Program of International Student Assessment (PISA), which assessed the math skills of 15-year-olds as a major domain and reading and science to a lesser extent. Mathematics was also assessed as a major domain in the base year 2003. PISA scores were normalized in the base year so that the score distribution had an average of 500 and a standard

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deviation of 100, which allows for the study of trends over a nine-year period. To understand the significance of the numerical data when considering trends and comparing scores among provinces, it should be noted that 41 points corresponds to the equivalent of one year of formal schooling (OECD 2014).

Between 2003 and 2012, all but two Canadian provinces showed statistically significant declines in the PISA math scores. Quebec's score remained constant over the period, and Saskatchewan experienced a decline that is not considered statistically significant. In some provinces, the decline was particularly steep: Manitoba and Alberta experienced dramatic declines of 36 and 32 points, respectively, and Manitoba, Newfoundland and Labrador, and Prince Edward Island scored below the 2012 OECD average (Richards 2014).

Students' PISA scores are reported along scales divided into six proficiency levels, with level 1 the lowest level and level 6 the highest. Level 2 is considered the baseline level to participate fully in society, while students who perform at levels 5 or 6 are considered to be capable of solving complex problems and have the skills necessary to compete successfully in a knowledge-based economy. Results from longitudinal studies have shown that students who perform below level 2 might be severely disadvantaged in their transition into higher education and into the labour force (OECD 2014). It is worrisome, therefore, that, in Alberta and Manitoba, the percentage of students who performed below level 2 doubled between 2003 and 2012, while the percentage who performed at or above level 5 nearly halved.[1]

Another international assessment, Trends in International Mathematics and Science Study (TIMSS), looks at the mathematics and science knowledge and skills of fourth and eighth graders. While PISA assesses mathematical literacy – mathematics problems that might be encountered in everyday life – TIMSS tests more specific skills such as knowledge of fractions and algebra, from both content and cognitive domains.

Alberta, Ontario and Quebec participated in the 2011 TIMSS assessment (Mullis et al. 2012). Quebec fared reasonably well internationally, but saw a decline of 34 points at the eighth grade level from its score in 1999. Alberta showed a decline of 26 points, similar to the decline in its PISA scores, while Ontario showed a slight increase from 1999 to 2003 and slight declines over the period from 2003 to 2011. Ontario has also seen declines in math scores on assessments administered by the Education Quality and Accountability Office (EQAO). Over a five-year period, the proportion of sixth graders who met Ontario provincial standards declined from 61 to 54 percent, and the proportion of third graders who met the standards fell from 71 to 67 percent.[2] Over the same period, provincial reading scores increased, suggesting that policies specific to mathematics contributed to the declines.

Domestically, the Council of Ministers of Education administers the Pan-Canadian Assessment of Science, Reading and Math (PCAP) triennially to Canadian eighth grade students.

1 In the 2003 assessment, 10.9 percent of Manitoba students performed below level 2 (compared with 21.2 percent in the 2012 assessment), while 7.4 percent of Alberta students performed below level 2 (compared with 15.1 percent in 2012). In the 2003 assessment, 18.9 percent of Manitoba students performed at levels 5 or 6 (compared with 10.3 percent in 2012), while 26.8 percent of Alberta students performed at levels 5 or 6 (compared with 16.9 percent in 2012).

2 Ontario's EQAO tests do not test basic skills since students are permitted calculators during the tests.

Mathematics was assessed as a major domain in 2010 (CMEC 2012) and as a minor domain in 2013, and scores improved in several provinces[3] – results that are hard to square with the declines in the PISA scores.[4] As a minor domain in 2013, however, the content coverage in mathematics was limited (CMEC 2014).

Emphasizing Best Teaching Practices

Best teaching practices in mathematics have been at the forefront of discussions regarding declining math scores in Canada, with a dichotomy emerging between two instructional techniques often referred to as discovery-based instruction and direct instruction. Proponents of either type of instruction generally agree that there are merits to using a balance of both approaches, but the specific balance is in dispute and the research evidence supporting effective teaching techniques must be considered.

Discovery-based Instruction: Changes in Canadian Classrooms

Discovery-based instruction has become popular in North America in recent years, making its way into Canadian curricula, teachers' professional development sessions and textbooks. A discovery-based learning environment often uses a top-down approach in which students are taught through problem solving or projects using hands-on objects. Discovery-based learning environments typically have some of the following characteristics:

- minimal guidance from the teacher and few explicit teacher explanations;
- open-ended problems with multiple solutions (Example: The answer to my question is 37. What might my question be?);
- frequent use of hands-on materials such as blocks, fraction strips and algebra tiles or drawing pictures to solve problems;
- use of multiple, preferably student-invented, strategies;
- minimal worksheet practice or written symbolic work;
- memorization of math facts is deprioritized;
- standard methods such as column addition or long division are downplayed;
- a top-down approach in which students work on complex problems, even though foundational skills might not be present.

Discovery-based instruction is not a new instructional technique. A review of literature since the 1950s shows that discovery-based instruction has often been repackaged under different names (Mayer 2004), such as inquiry-based instruction – which involves equivalent pedagogical techniques[5] – indirect instruction, problem-based learning, inquiry-based instruction, experiential learning and constructivist learning (Kirschner, Sweller, and Clark 2006). More recently, the term "twenty-first-century learning" has been used to describe many of these instructional techniques.

3 There were positive increases in mathematics scores on PCAP in British Columbia, Alberta, Saskatchewan, Quebec, Nova Scotia, Prince Edward Island and Newfoundland and Labrador, with the most noticeable increase in Prince Edward Island, where the mean score in mathematics increased from 460 to 492 between 2010 and 2013.

4 The Canadian average in mathematics on the 2013 PCAP assessment was 507. At the high end, Quebec students had a mean score of 527; students in the lowest-scoring province, Manitoba, had a mean score of 471.

5 These phrases are often interchanged to avoid criticism of certain pedagogical techniques, an approach that was recently used in Alberta. After a well-informed journalist for the *Edmonton Journal* wrote about the lack of evidence for discovery-based instruction, education officials argued that Alberta Education was actually promoting inquiry-based learning (Staples 2014).

The more conventional instructional approach is often called direct or explicit instruction. In this setting, students are directly taught concepts and given explicit explanations, followed by plenty of student practice, often with paper and pencil, feedback from the teacher and conventional assessment. Standard methods like column addition and long division are emphasized and students are encouraged to memorize basic facts like times tables. Direct instruction often follows a bottom-up approach in which students are taught foundational skills that are practiced to mastery, gradually preparing them for complex problem solving. Note, however, that the teaching of understanding – or why mathematical procedures and rules hold – can be encouraged in any instructional setting, including during direct instruction.

Shifting Curriculum

The first shift toward a discovery-based curriculum occurred in most Canadian provinces in the late 1990s and gathered momentum in the 2000s. Ontario adopted a new discovery-based math curriculum in 2005, and, since 2006, the western provinces and territories and the Maritime provinces have followed a curriculum designed by the Western National Curriculum Protocol (WNCP). These curricula saw expectations concerning outcomes such as fraction arithmetic shifted to later grades. At the same time, curricula and accompanying textbooks added methods with an emphasis on multiple strategies and hands-on materials such as blocks, fraction strips and algebra tiles.[6]

Proponents of discovery-based instruction argue that students learn better, have greater understanding and are less likely to forget information they discover themselves instead of being told the same information by an expert teacher. It is often claimed that direct instruction inhibits understanding and that, to become effective problem solvers, students must be exposed to rich problem-solving settings, which require them to develop their own techniques, instead of being taught problem-solving techniques explicitly. Those who favour discovery-based instruction are also concerned that the repetitive work required to memorize basic facts such as times tables might obstruct deeper understanding or cause math anxiety. Since most educators want instruction to cultivate students who understand mathematical concepts deeply, enjoy math, are able to transfer learning to new situations and are strong problem solvers, these claims sound appealing and have led to the shift toward discovery-based education in Canada.

The Role of Long-term Memory in Developing Math Skills

Despite these claims, the research evidence strongly favours direct instruction over discovery-based instruction for nurturing understanding, deeper learning and better problem solvers. To be effective, instructional techniques must cater to the limitations of a person's working memory, which can hold only a limited amount of new information. This is particularly important for novice learners who have difficulty focusing on new concepts when their working memory is overwhelmed. Learning occurs when information is transferred from working memory to long-term memory to be used later (Kirschner, Sweller, and Clark 2006).

When we encounter new information, it is stored in our very limited working memory, and

6 Provincially approved textbooks reinforce convoluted techniques such as drawing pictures to solve division problems such as 744 ÷ 6, and multiple strategies to teach single-digit multiplication problems such as 7 × 8 (Appel et al. 2008).

it is generally lost within about 20 seconds if not rehearsed. When information in our working memory is sufficiently practiced, it is then committed to long-term memory, after which it may be recalled later. An expert in mathematics stores a wealth of information in long-term memory, acquired through hours of experience and practice; when a new problem is encountered, knowledge and techniques are recalled from long-term memory to solve it (Hattie and Yates 2014). For instance, a seven-digit phone number is likely to be quickly forgotten unless it is repeated several times. There seems to be no limit to the amount of information that can be held in long-term memory – while an individual is likely incapable of storing more than one seven-digit phone number in working memory, many such numbers might be committed to long-term memory for later recall.

Transferring number facts such as times tables to long-term memory is another, more relevant example. Recent educational trends downplay the importance of times table memorization; students instead are encouraged to use a variety of strategies for determining multiplication facts (Ontario 2005; WNCP 2006). For example, students are often encouraged to work out number facts such as 7 × 8 using doubling: 8 is the double of 4 and 7 × 4 =28, so 7 × 8 = 28 + 28 = 56. Now consider the following problem. A storekeeper purchases 68 boxes of oranges at $27 per box. How much does he pay? The calculation 68 × 27 can be broken into bite-sized chunks that involve single-digit number facts such as 8 × 7 and 6 × 7, which should be accessible quickly from long-term memory.

Relying on ad hoc strategies to calculate the number facts involved, however, puts the student at an extreme disadvantage relative to one who can quickly recall a fact such as 7 × 8 = 56, since working memory becomes overburdened with the strategies involved to compute basic number facts. When they have committed times tables to long-term memory, students can concentrate on the more complex aspects of a problem. In the boxes of oranges example, that means translating word problems to mathematical statements, determining which operations to use and computing the two-digit multiplication.

The Evidence on Direct versus Discovery-based Instruction

Discovery methods ignore the limitations of working memory by eschewing conventional techniques such as times table memorization and by encouraging multiple, convoluted strategies instead of efficient, standard methods. Teaching through problem solving without providing the foundational skills necessary to solve problems overburdens working memory (Sweller 1988), and might not alter long-term memory, thereby inhibiting learning of mathematical concepts.

Controlled studies have shown that direct instruction is a much more effective teaching method: when learners are presented with new information, it should be explicitly taught by a teacher. Discovery-based learning environments often result in students becoming confused and frustrated and it is an inefficient style of instruction characterized by frequent false starts (Kirschner, Sweller, and Clark 2006). Numerous studies have found that, in contrast, direct instruction techniques such as worked examples, scaffolding, explicit explanations and consistent feedback are extremely beneficial for learning (Alfieri et al. 2011; Hattie and Yates 2014).

In a review of 200 research studies, Sutton Trust identifies key characteristics of effective instruction: teachers' use of assessment, reviewing previous learning, working through examples for students, giving adequate time for practice to embed skills securely in long-term memory, and introducing topics incrementally. The review also finds that less effective teachers often teach math using hands-on materials, delay the introduction of formal methods until they feel pupils are ready to move on, and encourage students to work things out for themselves using any method with which they feel comfortable (Sutton Trust 2014).

As well, studies consistently find that students who have difficulty with mathematics by the end of their primary school years have not memorized basic number facts, making further math learning difficult and resulting in feelings of helplessness and a lack of confidence and enjoyment (Hattie and Yates 2014). A great deal of time and effort is required to commit basic number facts to long-term memory, but the ability to recall them instantly frees up working memory, making it easier to learn new concepts.

Overemphasis on hands-on materials and pictures also presents problems. Although some materials, such as base-ten blocks, might assist initial learning, overuse can prevent the transfer of information to long-term memory, because working memory is assaulted with extraneous information. Transfer is more likely to occur if mathematical symbols are stressed over concrete materials (Kaminski, Sloutsky, and Heckler 2009).

A survey of eighth grade Canadian students who participated in the nationally administered 2010 PCAP, which tested mathematics as a major domain, shows a pattern similar to the international studies cited above. Students were asked to report the frequency with which their teachers used both indirect instruction methods and direct instructional techniques.[7] The use of direct instruction was positively correlated with better math performance for most students, except the highest achievers, who seemed to succeed regardless of the instructional method used. Furthermore, greater use of indirect instruction was found to be strongly associated with lower scores (CMEC 2012).

Best Methods to Teach Problem Solving and Understanding

Equipping students with strong problem-solving skills is an important goal of math education, but Canadian curricula and prominent resources ignore what research in cognitive science reveals about how problem-solving skills are acquired. Experienced, effective problem solvers store organized techniques in long-term memory, which allows them to categorize new problems and implement effective strategies to solve them (Sweller and Cooper 1985). The best way to ensure that students are well positioned to solve new problems is to provide them a library of knowledge and techniques and to teach thinking skills through direct instruction (Hattie and Yates 2014).

A large body of evidence shows that direct instruction through worked examples followed by practice with problems similar to the worked examples respects working-memory limitations and improves problem-solving performance (Paas and van Gog 2006; Sweller and Cooper 1985). Gradually increasing the difficulty level of worked examples and practice problems results in the ability of students to transfer problem-solving skills to new situations. However, when students are presented with problems that they do not have the techniques to solve without reference to worked examples,

7 The PCAP survey characterized indirect instruction techniques by teachers' use of hands-on materials (base-ten blocks, colour tiles, geometric solids), use of computer software, working in groups on investigations or problems, sharing solutions with other students in the class, and having opportunities to reflect on what was learned. Direct instruction techniques were characterized by conventional teaching methods: watching the teacher do examples, listening to the teacher give explanations, copying notes given by the teacher, practising new skills, undertaking teacher-guided investigations, reviewing skills learned, solving problems and working individually on investigations or problems.

they might struggle for long periods and learn little (Kirschner, Sweller, and Clark 2006). This also has an obvious negative effect on students' confidence.

As well, discovery-based learning does not lead to a better understanding of concepts or a higher quality of learning than direct instruction. On the contrary, Klahr and Nigam (2004) find that direct instruction results in much more learning than discovery-based instruction, and that students who learn in a direct instruction environment are no less proficient at translating learning to new situations. A particularly disturbing finding, from a number of studies, is that low-aptitude students perform worse on post-test measures after receiving discovery-based instruction than they do on pre-test measures. In other words, discovery-based instruction might result in learning losses and widen the gap between low- and high-performing students (Clark 1989).

What to Do?

Maintaining a balance between various instructional techniques is important, yet teachers want to use techniques that have proved to be effective. To that end, they should be made aware of controlled research studies that consistently find that direct instruction is much more effective than discovery-based instruction. Accordingly, teachers who wish to use discovery-based teaching techniques should consider doing so conservatively; the instructional balance should always favour direct instruction.

One way to redress the balance between instructional techniques that are effective and those that are less so would be to follow an 80/20 rule whereby at least 80 percent of instructional time is devoted to direct instructional techniques and 20 percent of instructional time (at most) favours discovery-based techniques. Although some individual teachers already might follow a roughly 80/20 rule, provincial curricula, teachers' professional development sessions and provincially approved (or mandated) textbooks tend to favour discovery-based techniques. Thus, pedagogical directives that stress ineffective discovery techniques should be removed from the curricula, and texts that incorporate effective direct instructional techniques should be included in provincially recommended textbook lists.

Emphasizing Concepts that Lead to Student Success

The cumulative nature of mathematics knowledge is often misunderstood or underestimated. Students cannot learn to multiply if they cannot add, they cannot add fractions if they are unable to work efficiently with whole numbers, and they cannot master algebra if they have not mastered fraction arithmetic. Delaying the introduction of important concepts in elementary and middle-years curricula results not only in a packed high-school curriculum, but also in students who are not prepared for success at that level. Experts in mathematics thus need to be involved in curriculum design, which, instead of considering concepts in isolation, must see curriculum outcomes in terms of where they will lead students.

Preparing Students for Success in Algebra

The US National Math Advisory Panel (NMAP 2008), which reviewed over 16,000 math education research articles, carefully considered outcomes that contribute to later success in mathematics, with particular attention to concepts and skills that prepare students for success in algebra. Algebra is the bloodline of high-school mathematics and the bridge to higher-level mathematics. Moreover, students who complete high-school algebra are twice as likely to graduate from college than those with less mathematical preparation, and completion of high-school algebra significantly correlates with higher earnings from employment (NMAP 2008). Countries whose students performed exceptionally well on the 2012 PISA assessment also stress algebra in school (OECD 2014).

In reviewing curricula from the highest-performing countries on the TIMSS assessment (Singapore, Japan, South Korea, Hong Kong, the Flemish part of Belgium and the Czech Republic) and surveying teachers of introductory algebra, the NMAP identified concepts and skills called Critical Concepts of Algebra that are necessary for success in algebra: fluency with whole numbers,[8] fluency with fractions,[9] and particular aspects of geometry and measurement.[10] The NMAP also provided benchmarks for what it calls "Critical Foundations" (see Table 1). These recommendations should not be taken in isolation. Whole number arithmetic, fraction arithmetic, geometric techniques and algebra should be the main focus, but they should also be taught through effective instruction, which includes explicit explanations by a teacher, followed by practice leading to student mastery.

The Importance of Fraction Arithmetic

Recent studies confirm the importance of these topics in early and middle years math curricula. There is plenty of evidence that automatic recall of number facts is extremely important for success in math (Price et al. 2013; Qin et al. 2014). Studies confirm – after controlling for various factors such as general intellectual ability, working memory, family income and education – that fluency with fraction arithmetic, without the use of calculators or technology, in early and middle-years students predicts student knowledge of algebra and later high-school math achievement (Bailey et al. 2012; Siegler et al. 2012).

Yet, most Canadian curricula delay the introduction of fraction arithmetic until the seventh and eighth grades. For example, in Manitoba, addition and subtraction of fractions, once covered in grades 4 and 5 (Manitoba 1978) were moved to grades 7 and 8 with that province's implementation of the WNCP curriculum in 1995. Similarly, in Ontario, fraction addition and subtraction is not covered until grade 7. Despite their importance for later success in math, the explicit expectation that times tables be memorized and standard algorithms be mastered[11] is absent from most Canadian curricula, and, in those curricula where these requirements are listed, they appear at later grades than those recommended by the NMAP.[12]

In the 2011 TIMSS assessment, eighth grade students from Ontario, Alberta and Quebec performed only slightly better than random guessing on questions that tested skill and understanding of fractions (see Figure 1). In contrast, in high-performing countries such as South Korea and Singapore, students handled fraction problems much more successfully.

8 This includes place value, computational fluency and knowledge of how to apply computational fluency to problem solving, memorization of math facts and fluency with standard algorithms for addition, subtraction, multiplication, division and estimation (NMAP 2008).

9 This includes carrying out operations with fractions confidently and efficiently, understanding why and how (finite) decimal numbers are fractions and the meaning of percent, the use of symbolic notation and the concept of generality (NMAP 2008).

10 This includes similar triangles, the slope of a straight line and finding unknown lengths, angles and areas (NMAP 2008).

11 The standard algorithms for arithmetic include vertical addition with carrying, subtraction with borrowing, the standard vertical array for multiplication and long division.

12 In response to public outcry, the Manitoba and Alberta governments recently added a curriculum requirement for times table memorization by the end of Grade 5. Manitoba also added standard algorithms, and Alberta eliminated the requirement that multiple strategies be taught. Alberta is in the process of a complete curriculum rewrite, which might see a move away from the WNCP curriculum, but might also still promote discovery or inquiry-based techniques.

Table 1: Benchmarks for the Critical Foundations of Mathematics

Fluency With Whole Numbers

1) By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
2) By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

Fluency With Fractions

1) By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
2) By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals.
3) By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
4) By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
5) By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
6) By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

Geometry and Measurement

1) By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
2) By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
3) By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Source: NMAP (2008), table 2.

Rewriting Curricula to Stress Important Concepts

To summarize, there are three main difficulties with Canadian math curricula. One is the explicit overemphasis on hands-on materials and models, which actually might hinder learning. Another is the stress on a multiple-strategy approach, which is time consuming, results in the overburden of working memory and the failure of students to master efficient techniques. A third difficulty is that important concepts are introduced too late.

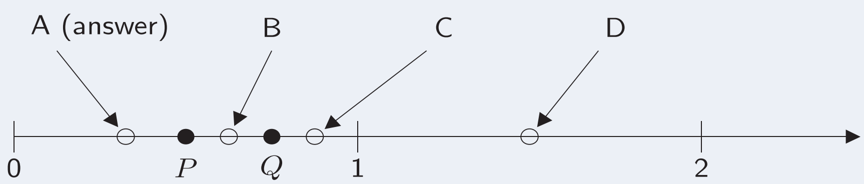
Accordingly, the curricula should be revised, using the NMAP's benchmarks for the Critical Foundations as a guideline, to ensure that concepts important for success are introduced at appropriate grade levels to give students plenty of opportunity

Figure 1: Two Grade 8 TIMSS Questions (2011)

I. (Basic arithmetic with fractions) Which method will find $\frac{1}{3} - \frac{1}{4}$?

$A: \frac{1-1}{4-3}$ $B: \frac{1}{4-3}$ $C: \frac{3-4}{3 \times 4}$ $D: \frac{4-3}{3 \times 4}$ (answer)

II. (Understanding multiplication) Fractions P and Q are shown on a number line. Which is the correct location of $N = P \times Q$?



Source: Adapted by Robert Craigen from Mullis et al. (2012).

Success Rates

System	I. Fractions	II. Multiplication
KOREA	86	44
SINGAPORE	83	45
TAIPEI	82	53
HONG KONG	77	47
QUEBEC	33	29
ONTARIO	33	27
ALBERTA	28	24
WORLD	37	23
GUESSING	25	25

Source: Compiled by Robert Craigen from Mullis et al. (2012).

to work with them and to consolidate appropriate techniques into long-term memory, with particular attention paid to whole number arithmetic, fraction arithmetic and algebra. Professional mathematicians, who have an overarching view of topics in the school curriculum as well as those beyond, should be included in curriculum revision teams to ensure mathematical rigour and accuracy and to ensure that concepts are not treated in isolation.

Improving Teachers' Content Knowledge in Mathematics

Teachers' fluency with topics of instruction underpins the successful translation of knowledge to students. Research confirms that teachers' math content knowledge is positively correlated with students' achievement in math, and that teachers should have a firm understanding of the math content they are required to teach and its connections to other math concepts, including those beyond the level teachers are assigned to teach (Ma 1999; NMAP 2008).

For example, a study of first and third grade teachers and their students found that the difference in learning gains between students of high- and low-scoring teachers on a test of "Content Knowledge for Teaching" – defined as mathematical knowledge that is relevant to teaching – is similar to the strength of the relationship between socioeconomic background and academic attainment (Hill, Rowan, and Ball 2005). In other words, teachers' math content knowledge positively affects students' math achievement to the same extent that socioeconomic background affects achievement in math, even in the lowest grades. Thus, an elementary school teacher should be able to complete computations with whole numbers and fractions efficiently, for instance, and to provide correct explanations for why mathematical procedures work and to evaluate the validity of mathematical arguments.

Teachers in countries where students perform well on international math assessments tend to have high scores on math knowledge tests. Moreover, those countries tend to emphasize policies that ensure the high quality of entrants into teacher education programs, high standards for teacher licensing after graduation, a balance between teacher supply and demand and rigorous systems of assessment and accreditation of teacher education programs (Ingvarson et al. 2013; Schmidt, Houang, and Cogan 2011).

In Canada, high-school math teachers are often specialists with qualifications in math, but elementary school teachers are most often generalists who are assigned to teach a variety of subjects, including mathematics. In some provinces, middle-years teachers – those who teach seventh and eighth grades – are also generalists. Math course requirements for prospective teachers vary significantly across Canada. In Manitoba and Alberta, for instance, elementary and middle-years teachers are required to complete one three-credit-hour mathematics course in university before being licensed to teach. In Quebec, where students outperform those in other provinces on math assessments, elementary teachers are generally required to complete two three-credit-hour mathematics courses in university, while middle-years math teachers are often specialists who have completed as many as 15 three-credit-hour university math courses. In Ontario, the completion of a university math course is recommended, but not provincially mandated, for pre-service elementary teachers, although the Ontario government has announced funding for classroom teachers to upgrade their mathematics skills (Ontario 2014).

The Solution: Math Content Courses in University and Math Licensure Exams

It is imperative that policies be adopted to ensure that future elementary and middle-years teachers have a deep understanding of the mathematics they are expected to teach prior to being certified. This could be accomplished through a combination of course requirements for teacher candidates and mandatory provincial licensure exams that assess mathematics knowledge and skill.

In provinces that follow the early and middle-years generalist model, I recommend that future elementary teachers be required to complete a minimum of six credit hours of university mathematics content courses prior to obtaining provincial certification. Since teacher preparation programs, entrance requirements for university faculties of education and math content courses for prospective early and middle-years teachers vary significantly across provinces and among different universities within provinces, this recommendation on its own might not be sufficient.

Many professions require would-be entrants to pass an exam prior to receiving a licence to practice. Thus, to ensure consistency and to achieve a high level of mathematics content knowledge on the part of elementary and middle-years math teachers, provincial governments should consider instituting licensure exams that assess proficiency in the mathematics that a teaching candidate will be certified to teach, with a focus on topics, such as fractions and algebra, that correlate with students' success in later mathematics. Such exams should require candidates to demonstrate skills and fluent computational techniques from the elementary and middle-years math curriculum without the use of calculators or hands-on materials, and to provide accurate explanations of common mathematical procedures and rules. Licensure exams should be free of pedagogy, and should be designed by experts in mathematics to ensure rigour and accuracy.

Conclusion

The importance of mathematics in a knowledge- and technology-based economy, the correlation between early math achievement and later academic success and the decline in Canadian students' mathematics scores on international tests in recent years suggest that provincial governments would be wise to improve the way mathematics is taught in Canadian schools.

Recent shifts in math teaching practices coupled with radical, discovery-based math curricula are seriously hampering math learning by Canadian students. Evidence shows that direct instructional techniques work better than discovery-based techniques, so teachers should follow an 80/20 rule, devoting at least 80 percent of their math instructional time to direct instructional techniques. Curricula also should be revised to remove ineffective instructional directives, and streamlined to focus on explicit topics and concepts that have been shown to predict later success in math. As well, the need to improve the math content knowledge of future elementary and middle-years math teachers should be addressed through course requirements and licensure exams.

Adopting these recommendations should help solve some of the root problems behind the falling math scores of Canadian students, and result in improvements in the years to come.

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