

# Mathematics



# Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21<sup>st</sup> century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at [www.corestandards.org](http://www.corestandards.org). While all the expectations of the Common Core State Standards are embedded and **coded with CCSS**: in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

# Overview of Changes Mathematics Standards

## Principles of the Standards Review Process

The Colorado Model Content Standards revision process was informed by these guiding principles:

- Begin with the end in mind; define what prepared graduates need to be successful using 21<sup>st</sup> century skills in our global economy.
- Align K-12 standards with early childhood expectations and higher education.
- Change is necessary.
- Standards will be deliberately designed for clarity, rigor, and coherence.
- There will be fewer, higher and clearer standards.
- Standards will be actionable.

## Notable Changes to the Colorado Model Content Standards in Mathematics

The most evident changes to the Colorado standards are replacing grade-band expectations (K-4, 5-8, and 9-12) with grade-level specific expectations. These are explained here in addition to other changes that are apparent upon comparison between the current mathematics standards and the proposed changes.

1. **Impact of standards articulation by grade level.** The original Colorado Model Content Standards for Mathematics were designed to provide districts with benchmarks of learning at grades 4, 8, and 12. The mathematics standards revision subcommittee was charged with providing more a specific learning trajectory of concepts and skills across grade levels, from early school readiness to postsecondary preparedness. Articulating standards by grade level up to eighth grade in mathematics affords greater specificity (clearer standards) in describing the learning path across levels (higher standards), while focusing on a few key ideas at each grade level (fewer standards).
2. **Articulation of high school standards.** High school standards are not articulated by grade level, but by standard. This is intended to support district decisions about how best to design curriculum and courses – whether through an integrated approach, a traditional course sequence, or alternative approaches such as career and technical education. The high school mathematics standards delineate what all high school students should know and be able to do in order to be well prepared for any postsecondary option. The individual standards are not meant to represent a course or a particular timeframe. All high school students should be able to reach these rigorous standards within four years. Students with advanced capability may accomplish these expectations in a shorter timeframe leaving open options for study of other advanced mathematics.
3. **Integration of P-2 Council’s recommendations.** The mathematics subcommittee integrated the *Building Blocks to the Colorado K-12 Content Standards* document into the P-12 mathematics standards, aligning expectations to a great degree. Important mathematics concepts and skills are defined clearly across these foundational years, detailing expectations to a much greater extent for teachers and parents.
4. **Standards are written for mastery.** The proposed revisions to standards define mastery of concepts and skills. Mastery means that a student has facility with a skill or concept in multiple contexts. This is not an indication that instruction at a grade-level expectation begins and only occurs at that grade level. Maintenance of previously mastered concepts and skills and scaffolding future learning are the domain of curriculum and instruction – not standards.

5. **Integration of the Common Core State Standards.** These revised standards reflect the inclusion of the Common Core State Standards in Mathematics.
6. **The processes and procedures of school Algebra have been made more explicit.** More specificity about algebraic procedures is apparent in the Patterns, Functions and Algebraic Structures expectations.

For instance, two high school expectations read:

- Expressions, equations, and inequalities can be expressed in multiple, equivalent forms.
  - Solutions to equations, inequalities and systems of equations are found using a variety of tools.
7. **Emphasis on concepts and skills across grade levels.** The subcommittee deliberately designed the standards to emphasize specific concepts and skills at different grade levels. This allows teachers to focus on fewer concepts at greater depth than in the past.
  8. **Integration of technology, most notably at the high school level.** The standards integrate appropriate technology to allow students access to concepts and skills in mathematics in ways that mirror the 21<sup>st</sup> century workplace.
  9. **Greater focus on Data Analysis, Statistics, and Probability at the middle and high school levels.** Information literacy in mathematics involves the ability to manage and make sense of data in more sophisticated ways than in the past. This involves emphasizing Data Analysis, Statistics, and Probability to a greater degree than in the original mathematics standards.
  10. **Intentional integration of personal financial literacy (PFL).** Personal financial literacy was integrated preschool through grade twelve in the math standards in order to assure high school graduates are fiscally responsible. House Bill 08-1168 requires standards which includes these skills: goal setting, financial responsibility, income and career; planning, saving and investing, using credit; risk management and insurance.

Below is a quick guide to other changes in the mathematics standards:

Area		Summary of changes	
	<b>2005 Colorado Model Content Standards</b>	<b>2010 Colorado Academic Standards</b>	
<b>Number of standards</b>	Colorado has six standards in mathematics	Combine current standards 1 and 6 and standards 4 and 5. There are now four standards	
<b>Names of standards</b>	<p><b>Standard 1</b> Number Sense and Number Relationships</p> <p><b>Standard 2</b> Patterns and Algebra</p> <p><b>Standard 3</b> Data and Probability</p> <p><b>Standard 4</b> Geometry</p> <p><b>Standard 5</b> Measurement</p> <p><b>Standard 6</b> Computation</p>	<p><b>Standard 1</b> Number Sense, Properties, and Operations</p> <p><b>Standard 2</b> Patterns, Functions, and Algebraic Structures</p> <p><b>Standard 3</b> Data Analysis, Statistics, and Probability</p> <p><b>Standard 4</b> Shape, Dimension, and Geometric Relationships</p>	
<b>Integration of 21<sup>st</sup> century and postsecondary workforce readiness skills</b>	<ul style="list-style-type: none"> <li>Not deliberately addressed in original document.</li> </ul>	<ul style="list-style-type: none"> <li>A design feature of the revision process.</li> <li>Intentionally integrated into evidence outcomes.</li> </ul>	
<b>P-2</b>	<ul style="list-style-type: none"> <li>Standards articulated for grade band beginning with kindergarten.</li> <li>Benchmarks articulated by grade band of K-4 with most geared to upper grades.</li> </ul>	<ul style="list-style-type: none"> <li>Pre-K included.</li> <li>Grade level expectations articulated for each elementary grade.</li> <li>Clear expectations articulated for grades P-2.</li> </ul>	
<b>Number of grade level expectations (GLE)</b>	<ul style="list-style-type: none"> <li>Average of 27 benchmarks per grade level.</li> </ul>	<ul style="list-style-type: none"> <li>Average of 7 grade level expectations per grade level (K-8), with 14 for high school.</li> </ul>	
<b>Integration of Personal Financial Literacy (PFL)</b>	<ul style="list-style-type: none"> <li>Not deliberately addressed in original document.</li> </ul>	<ul style="list-style-type: none"> <li>A design feature of the revision process.</li> <li>Intentionally integrated into evidence outcomes.</li> </ul>	

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## **Mathematics National Expert Reviewer**

Dr. Ann Shannon is a mathematics educator with many decades of experience who specializes in standards, assessment, and curriculum. Currently, Shannon works as consultant helping states, districts, and schools to better serve the needs of diverse learners of mathematics.

Dr. Shannon was employed as a research fellow at the Shell Centre for Mathematics Education, University Nottingham, England before moving to the University of California, Berkeley in 1994.

At the University of California, she developed performance assessments for the NSF-funded Balanced Assessment project and the New Standards project. Her 1999 monograph, *Keeping Score*, was published by the National Research Council and drew on her work for Balanced Assessment and New Standards.

Recently Shannon has helped Maine, Georgia, and Rhode Island develop academic standards for learning mathematics.

## References

The mathematics subcommittee used a variety of resources representing a broad range of perspectives to inform its work. Those references include:

- ***Singapore National Curriculum***
- ***Massachusetts Curriculum Framework***
- ***Virginia Standards of Learning***
- ***Finland – National Core Curriculum***
- WestEd Colorado Model Content Standards Review
- Achieve *Benchmarks for Elementary, Middle, and High School Mathematics*
- Benchmarks 2061
- College Board *Standards for College Success*
- ***Guidelines for Assessment and Instruction in Statistics Education (GAISE)***
- ***NCTM Principles and Standards for School Mathematics and Focal Points***
- ***Standards for Success "Understanding University Success"***
- Minnesota Academic Standards, Mathematics K-12
- Building Blocks to the Colorado K-12 Content Standards
- National Math Panel Report

## Colorado Academic Standards Mathematics Standards

*"Pure mathematics is, in its way, the poetry of logical ideas."*

*Albert Einstein*

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*"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."*

*Dr. Michael Brown, Nobel Prize Laureate*

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In the 21<sup>st</sup> century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

## Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21<sup>st</sup> century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The “Continuum of State Standards Definitions” section that follows shows the hierarchical order of the standards components. The “Standards Template” section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

**Prepared Graduate Competencies:** The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

**Standard:** The topical organization of an academic content area.

**High School Expectations:** The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

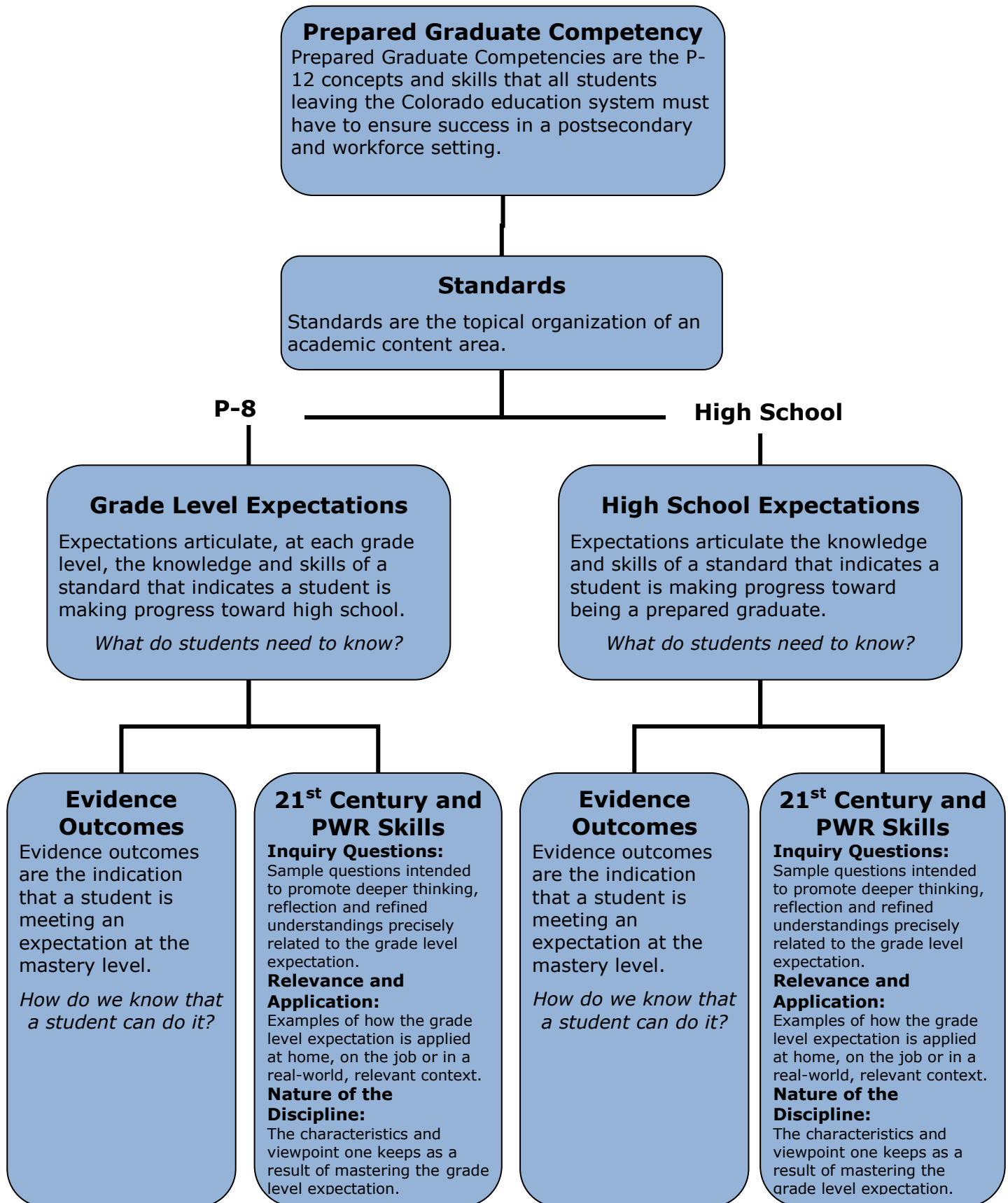
**Grade Level Expectations:** The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

**Evidence Outcomes:** The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

**21<sup>st</sup> Century Skills and Readiness Competencies:** Includes the following:

- ***Inquiry Questions:***  
Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
- ***Relevance and Application:***  
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
- ***Nature of the Discipline:***  
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

## Continuum of State Standards Definitions



## STANDARDS TEMPLATE

**Content Area: NAME OF CONTENT AREA**

**Standard:** The topical organization of an academic content area.

**Prepared Graduates:**

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

### High School and Grade Level Expectations

**Concepts and skills students master:**

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

*What do students need to know?*

**Evidence Outcomes**

**Students can:**

Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

*How do we know that a student can do it?*

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

**Relevance and Application:**

Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

**Nature of the Discipline:**

The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

## Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

## Colorado Academic Standards Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

### **1. Number Sense, Properties, and Operations**

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

### **2. Patterns, Functions, and Algebraic Structures**

Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

### **3. Data Analysis, Statistics, and Probability**

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

### **4. Shape, Dimension, and Geometric Relationships**

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

### **Modeling Across the Standards**

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (\*).



## **Standards for Mathematical Practice from The Common Core State Standards for Mathematics**

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

### **Mathematics | Standards for Mathematical Practice**

*The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).*

#### **1. Make sense of problems and persevere in solving them.**

*Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.*

#### **2. Reason abstractly and quantitatively.**

*Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.*

#### **3. Construct viable arguments and critique the reasoning of others.**

*Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.*

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

#### **7. Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or

as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>High School</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>1. The complex number system includes real numbers and imaginary numbers</li> <li>2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations</li> </ol>
2. Patterns, Functions, and Algebraic Structures	<ol style="list-style-type: none"> <li>1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables</li> <li>2. Quantitative relationships in the real world can be modeled and solved using functions</li> <li>3. Expressions can be represented in multiple, equivalent forms</li> <li>4. Solutions to equations, inequalities and systems of equations are found using a variety of tools</li> </ol>
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>1. Visual displays and summary statistics condense the information in data sets into usable knowledge</li> <li>2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions</li> <li>3. Probability models outcomes for situations in which there is inherent randomness</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically</li> <li>2. Concepts of similarity are foundational to geometry and its applications</li> <li>3. Objects in the plane can be described and analyzed algebraically</li> <li>4. Attributes of two- and three-dimensional objects are measurable and can be quantified</li> <li>5. Objects in the real world can be modeled using geometric concepts</li> </ol>

From the Common State Standards for Mathematics, Pages 58, 62, 67, 72-74, and 79.

### **Mathematics | High School—Number and Quantity**

**Numbers and Number Systems.** *During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.*

*With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.*

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that  $(5^{1/3})^3$  should be  $5^{(1/3)3} = 5^1 = 5$  and that  $5^{1/3}$  should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

**Quantities.** In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

### **Mathematics | High School—Algebra**

**Expressions.** An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example,  $p + 0.05p$  can be interpreted as the addition of a 5% tax to a price  $p$ . Rewriting  $p + 0.05p$  as  $1.05p$  shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example,  $p + 0.05p$  is the sum of the simpler expressions  $p$  and  $0.05p$ . Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

**Equations and inequalities.** An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of  $x + 1 = 0$  is an integer, not a whole number; the solution of  $2x + 1 = 0$  is a rational number, not an integer; the solutions of  $x^2 - 2 = 0$  are real numbers, not rational numbers; and the solutions of  $x^2 + 2 = 0$  are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid,  $A = ((b_1 + b_2)/2)h$ , can be solved for  $h$  using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

**Connections to Functions and Modeling.** Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

### **Mathematics | High School—Functions**

Functions describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour,  $v$ ; the rule  $T(v) = 100/v$  expresses this relationship algebraically and defines a function whose name is  $T$ .

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like  $f(x) = a + bx$ ; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

### **Connections to Expressions, Equations, Modeling, and Coordinates.**

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the

same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

### **Mathematics | High School—Modeling**

*Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.*

*A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.*

*Some examples of such situations might include:*

- *Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.*
- *Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.*
- *Designing the layout of the stalls in a school fair so as to raise as much money as possible.*
- *Analyzing stopping distance for a car.*
- *Modeling savings account balance, bacterial colony growth, or investment growth.*
- *Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.*
- *Analyzing risk in situations such as extreme sports, pandemics, and terrorism.*
- *Relating population statistics to individual predictions.*

*In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.*

*One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.*

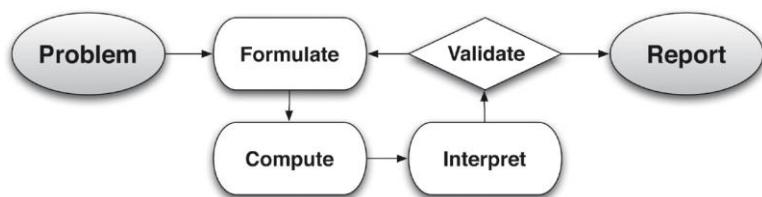
*The basic modeling cycle is summarized in the diagram (below). It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.*

*In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO<sub>2</sub> over time.*

*Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.*

*Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.*

**Modeling Standards.** *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (\*).*



### **Mathematics | High School—Geometry**

*An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.*

*Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)*

*During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.*

*The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.*

*In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.*



*Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.*

*The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.*

*Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.*

*Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.*

**Connections to Equations.** *The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.*

### **Mathematics | High School—Statistics and Probability\***

*Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.*

*Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.*

*Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to*

*consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.*

*Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.*

*Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.*

**Connections to Functions and Modeling.** *Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Eighth Grade</b>	
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically 2. Properties of algebra and equality are used to solve linear equations and systems of equations 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity 2. Direct and indirect measurement can be used to describe and make comparisons

From the Common State Standards for Mathematics, Page 52.

### **Mathematics | Grade 8**

*In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.*

*(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ( $y/x = m$  or  $y = mx$ ) as special linear equations ( $y = mx + b$ ), understanding that the constant of proportionality ( $m$ ) is the slope, and the graphs are lines through the origin. They understand that the slope ( $m$ ) of a line is a constant rate of change, so that if the input or  $x$ -coordinate changes by an amount  $A$ , the output or  $y$ -coordinate changes by the amount  $m \cdot A$ . Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and  $y$ -intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.*

*(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and*

*graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.*

*(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Seventh Grade</b>	
1. Number Sense, Properties, and Operations	1. Proportional reasoning involves comparisons and multiplicative relationships among ratios 2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently
2. Patterns, Functions, and Algebraic Structures	1. Properties of arithmetic can be used to generate equivalent expressions 2. Equations and expressions model quantitative relationships and phenomena
3. Data Analysis, Statistics, and Probability	1. Statistics can be used to gain information about populations by examining samples 2. Mathematical models are used to determine probability
4. Shape, Dimension, and Geometric Relationships	1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof 2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

From the Common State Standards for Mathematics, Page 46.

### **Mathematics | Grade 7**

*In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.*

*(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve*

*a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.*

*(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.*

*(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the*

*relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.*

*(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Sixth Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>Quantities can be expressed and compared using ratios and rates</li> <li>Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency</li> <li>In the real number system, rational numbers have a unique location on the number line and in space</li> </ol>
2. Patterns, Functions, and Algebraic Structures	<ol style="list-style-type: none"> <li>Algebraic expressions can be used to generalize properties of arithmetic</li> <li>Variables are used to represent unknown quantities within equations and inequalities</li> </ol>
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>Objects in space and their parts and attributes can be measured and analyzed</li> </ol>

From the Common State Standards for Mathematics, Pages 39-40

### **Mathematics | Grade 6**

*In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.*

*(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.*

*(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.*

*(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve*

simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as  $3x = y$ ) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.



# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Fifth Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms</li> <li>2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency</li> <li>3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency</li> <li>4. The concepts of multiplication and division can be applied to multiply and divide fractions</li> </ol>
2. Patterns, Functions, and Algebraic Structures	<ol style="list-style-type: none"> <li>1. Number patterns are based on operations and relationships</li> </ol>
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>1. Visual displays are used to interpret data</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Properties of multiplication and addition provide the foundation for volume an attribute of solids</li> <li>2. Geometric figures can be described by their attributes and specific locations in the plane</li> </ol>

From the Common State Standards for Mathematics, Page 33.

### **Mathematics | Grade 5**

*In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.*

*(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)*

*(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.*

*(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Fourth Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms</li> <li>2. Different models and representations can be used to compare fractional parts</li> <li>3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency</li> </ol>
2. Patterns, Functions, and Algebraic Structures	<ol style="list-style-type: none"> <li>1. Number patterns and relationships can be represented by symbols</li> </ol>
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>1. Visual displays are used to represent data</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time</li> <li>2. Geometric figures in the plane and in space are described and analyzed by their attributes</li> </ol>

From the Common State Standards for Mathematics, Page 27.

### **Mathematics | Grade 4**

*In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.*

*(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.*

*(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $15/9 = 5/3$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.*

*(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Third Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>1. The whole number system describes place value relationships and forms the foundation for efficient algorithms</li> <li>2. Parts of a whole can be modeled and represented in different ways</li> <li>3. Multiplication and division are inverse operations and can be modeled in a variety of ways</li> </ol>
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>1. Visual displays are used to describe data</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Geometric figures are described by their attributes</li> <li>2. Linear and area measurement are fundamentally different and require different units of measure</li> <li>3. Time and attributes of objects can be measured with appropriate tools</li> </ol>

From the Common State Standards for Mathematics, Page 21.

### **Mathematics | Grade 3**

*In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.*

*(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.*

*(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.*

*(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By*

*decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.*

*(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Second Grade</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms</li> <li>Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency</li> </ol>
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	<ol style="list-style-type: none"> <li>Visual displays of data can be constructed in a variety of formats to solve problems</li> </ol>
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>Shapes can be described by their attributes and used to represent part/whole relationships</li> <li>Some attributes of objects are measurable and can be quantified using different tools</li> </ol>

From the Common State Standards for Mathematics, Page 17.

### **Mathematics | Grade 2**

*In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.*

*(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).*

*(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.*

*(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.*

*(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>First Grade</b>	
1. Number Sense, Properties, and Operations	1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms 2. Number relationships can be used to solve addition and subtraction problems
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	1. Visual displays of information can be used to answer questions
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be described by defining attributes and created by composing and decomposing 2. Measurement is used to compare and order objects and events

From the Common State Standards for Mathematics, Page 13.

### **Mathematics | Grade 1**

*In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.*

*(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.*

*(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.*

*(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.<sup>1</sup>*

*(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the*



*original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry*

*<sup>1</sup>Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Kindergarten</b>	
1. Number Sense, Properties, and Operations	<ol style="list-style-type: none"> <li>1. Whole numbers can be used to name, count, represent, and order quantity</li> <li>2. Composing and decomposing quantity forms the foundation for addition and subtraction</li> </ol>
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Shapes are described by their characteristics and position and created by composing and decomposing</li> <li>2. Measurement is used to compare and order objects</li> </ol>

From the Common State Standards for Mathematics, Page 9.

### **Mathematics | Kindergarten**

*In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.*

*(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as  $5 + 2 = 7$  and  $7 - 2 = 5$ . (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.*

*(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.*

# Mathematics

## Grade Level Expectations at a Glance

Standard	Grade Level Expectation
<b>Preschool</b>	
1. Number Sense, Properties, and Operations	1. Quantities can be represented and counted
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	<ol style="list-style-type: none"> <li>1. Shapes can be observed in the world and described in relation to one another</li> <li>2. Measurement is used to compare objects</li> </ol>

## **21<sup>st</sup> Century Skills and Readiness Competencies in Mathematics**

Mathematics in Colorado's description of 21<sup>st</sup> century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21<sup>st</sup> century skills, as follows:

### Critical Thinking and Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

### Information Literacy

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

### Collaboration

Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

### Self-Direction

Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

### Invention

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

## **Colorado’s Description for School Readiness**

*(Adopted by the State Board of Education, December 2008)*

School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

## **Colorado’s Description of Postsecondary and Workforce Readiness**

*(Adopted by the State Board of Education, June 2009)*

Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

## **How These Skills and Competencies are Embedded in the Revised Standards**

Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline*. These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

**Inquiry Questions** – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

**Relevance and Application** – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

**Nature of Discipline** – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind’s eye.

# 1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

## **Prepared Graduates**

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

### **Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:**

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: High School

##### Concepts and skills students master:

1. The complex number system includes real numbers and imaginary numbers

##### Evidence Outcomes

###### Students can:

- a. Extend the properties of exponents to rational exponents. (CCSS: N-RN)
  - i. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.<sup>1</sup> (CCSS: N-RN.1)
  - ii. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: N-RN.2)
- b. Use properties of rational and irrational numbers. (CCSS: N-RN)
  - i. Explain why the sum or product of two rational numbers is rational. (CCSS: N-RN.3)
  - ii. Explain why the sum of a rational number and an irrational number is irrational. (CCSS: N-RN.3)
  - iii. Explain why the product of a nonzero rational number and an irrational number is irrational. (CCSS: N-RN.3)
- c. Perform arithmetic operations with complex numbers. (CCSS: N-CN)
  - i. Define the complex number  $i$  such that  $i^2 = -1$ , and show that every complex number has the form  $a + bi$  where  $a$  and  $b$  are real numbers. (CCSS: N-CN.1)
  - ii. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: N-CN.2)
- d. Use complex numbers in polynomial identities and equations. (CCSS: N-CN)
  - i. Solve quadratic equations with real coefficients that have complex solutions. (CCSS: N-CN.7)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. When you extend to a new number systems (e.g., from integers to rational numbers and from rational numbers to real numbers), what properties apply to the extended number system?
2. Are there more complex numbers than real numbers?
3. What is a number system?
4. Why are complex numbers important?

###### Relevance and Application:

1. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane.

###### Nature of Mathematics:

1. Mathematicians build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems.
2. Mathematics involves making and testing conjectures, generalizing results, and making connections among ideas, strategies, and solutions.
3. Mathematicians look for and make use of structure. (MP)
4. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

<b>Prepared Graduates:</b> <ul style="list-style-type: none"><li>➤ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error</li></ul>
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**Grade Level Expectation: High School**

<b>Concepts and skills students master:</b> <ul style="list-style-type: none"><li>2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations</li></ul>
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<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<b>Students can:</b> <ul style="list-style-type: none"><li>a. Reason quantitatively and use units to solve problems (CCSS: N-Q)<ul style="list-style-type: none"><li>i. Use units as a way to understand problems and to guide the solution of multi-step problems. (CCSS: N-Q.1)<ul style="list-style-type: none"><li>1. Choose and interpret units consistently in formulas. (CCSS: N-Q.1)</li><li>2. Choose and interpret the scale and the origin in graphs and data displays. (CCSS: N-Q.1)</li></ul></li><li>ii. Define appropriate quantities for the purpose of descriptive modeling. (CCSS: N-Q.2)</li><li>iii. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: N-Q.3)</li><li>iv. Describe factors affecting take-home pay and calculate the impact (PFL)</li><li>v. Design and use a budget, including income (net take-home pay) and expenses (mortgage, car loans, and living expenses) to demonstrate how living within your means is essential for a secure financial future (PFL)</li></ul></li></ul>	<b>Inquiry Questions:</b> <ul style="list-style-type: none"><li>1. Can numbers ever be too big or too small to be useful?</li><li>2. How much money is enough for retirement? (PFL)</li><li>3. What is the return on investment of post-secondary educational opportunities? (PFL)</li></ul> <b>Relevance and Application:</b> <ul style="list-style-type: none"><li>1. The choice of the appropriate measurement tool meets the precision requirements of the measurement task. For example, using a caliper for the manufacture of brake discs or a tape measure for pant size.</li><li>2. The reading, interpreting, and writing of numbers in scientific notation with and without technology is used extensively in the natural sciences such as representing large or small quantities such as speed of light, distance to other planets, distance between stars, the diameter of a cell, and size of a micro-organism.</li><li>3. Fluency with computation and estimation allows individuals to analyze aspects of personal finance, such as calculating a monthly budget, estimating the amount left in a checking account, making informed purchase decisions, and computing a probable paycheck given a wage (or salary), tax tables, and other deduction schedules.</li></ul> <b>Nature of Mathematics:</b> <ul style="list-style-type: none"><li>1. Using mathematics to solve a problem requires choosing what mathematics to use; making simplifying assumptions, estimates, or approximations; computing; and checking to see whether the solution makes sense.</li><li>2. Mathematicians reason abstractly and quantitatively. (MP)</li><li>3. Mathematicians attend to precision. (MP)</li></ul>



**Standard: 1. Number Sense, Properties, and Operations**  
**High School**

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<sup>1</sup> For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5. (CCSS: N-RN.1)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Eighth Grade

##### Concepts and skills students master:

1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line

##### Evidence Outcomes

###### Students can:

- a. Define irrational numbers.<sup>1</sup>
- b. Demonstrate informally that every number has a decimal expansion. (CCSS: 8.NS.1)
  - i. For rational numbers show that the decimal expansion repeats eventually. (CCSS: 8.NS.1)
  - ii. Convert a decimal expansion which repeats eventually into a rational number. (CCSS: 8.NS.1)
- c. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.<sup>2</sup> (CCSS: 8.NS.2)
- d. Apply the properties of integer exponents to generate equivalent numerical expressions.<sup>3</sup> (CCSS: 8.EE.1)
- e. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. (CCSS: 8.EE.2)
- f. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.<sup>4</sup> (CCSS: 8.EE.2)
- g. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.<sup>5</sup> (CCSS: 8.EE.3)
- h. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. (CCSS: 8.EE.4)
  - i. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.<sup>6</sup> (CCSS: 8.EE.4)
  - ii. Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.4)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. Why are real numbers represented by a number line and why are the integers represented by points on the number line?
2. Why is there no real number closest to zero?
3. What is the difference between rational and irrational numbers?

###### Relevance and Application:

1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle.
2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density).
3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents.

###### Nature of Mathematics:

1. Mathematics provides a precise language to describe objects and events and the relationships among them.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

**Standard: 1. Number Sense, Properties, and Operations**  
**Eighth Grade**

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<sup>1</sup> Know that numbers that are not rational are called irrational. (CCSS: 8.NS.1)

<sup>2</sup> e.g.,  $n^2$ . (CCSS: 8.NS.2)

For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.2)

<sup>3</sup> For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ . (CCSS: 8.EE.1)

<sup>4</sup> Know that  $\sqrt{2}$  is irrational. (CCSS: 8.EE.2)

<sup>5</sup> For example, estimate the population of the United States as 3 times  $10^8$  and the population of the world as 7 times  $10^9$ , and determine that the world population is more than 20 times larger. (CCSS: 8.EE.3)

<sup>6</sup> e.g., use millimeters per year for seafloor spreading. (CCSS: 8.EE.4)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

#### Grade Level Expectation: Seventh Grade

##### Concepts and skills students master:

1. Proportional reasoning involves comparisons and multiplicative relationships among ratios

##### Evidence Outcomes

###### Students can:

- a. Analyze proportional relationships and use them to solve real-world and mathematical problems.(CCSS: 7.RP)
- b. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.<sup>1</sup> (CCSS: 7.RP.1)
- c. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.2)
  - i. Determine whether two quantities are in a proportional relationship.<sup>2</sup> (CCSS: 7.RP.2a)
  - ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.2b)
  - iii. Represent proportional relationships by equations.<sup>3</sup> (CCSS: 7.RP.2c)
  - iv. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate. (CCSS: 7.RP.2d)
- d. Use proportional relationships to solve multistep ratio and percent problems.<sup>4</sup> (CCSS: 7.RP.3)
  - i. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality (PFL)
  - ii. Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease (PFL)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison?
2. What comparisons can be made using ratios?
3. How do you know when a proportional relationship exists?
4. How can proportion be used to argue fairness?
5. When is it better to use an absolute comparison?
6. When is it better to use a relative comparison?

###### Relevance and Application:

1. The use of ratios, rates, and proportions allows sound decision-making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies.
2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories.
3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures.

###### Nature of Mathematics:

1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

**Content Area: Mathematics**  
**Standard: 1. Number Sense, Properties, and Operations**

<b>Prepared Graduates:</b>	
<ul style="list-style-type: none"> <li>➤ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency</li> </ul>	
<b>Grade Level Expectation: Seventh Grade</b>	
<b>Concepts and skills students master:</b>	
2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently	
<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<b>Students can:</b> a. Apply understandings of addition and subtraction to add and subtract rational numbers including integers. (CCSS: 7.NS.1) <ul style="list-style-type: none"> <li>i. Represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.1)</li> <li>ii. Describe situations in which opposite quantities combine to make 0.<sup>5</sup> (CCSS: 7.NS.1a)</li> <li>iii. Demonstrate <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. (CCSS: 7.NS.1b)</li> <li>iv. Show that a number and its opposite have a sum of 0 (are additive inverses). (CCSS: 7.NS.1b)</li> <li>v. Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.1c)</li> <li>vi. Demonstrate subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. (CCSS: 7.NS.1c)</li> <li>vii. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.1c)</li> <li>viii. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.1d)</li> </ul> b. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers including integers. (CCSS: 7.NS.2) <ul style="list-style-type: none"> <li>i. Apply properties of operations to multiplication of rational numbers.<sup>6</sup> (CCSS: 7.NS.2a)</li> <li>ii. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.2a)</li> <li>iii. Apply properties of operations to divide integers.<sup>7</sup> (CCSS: 7.NS.2b)</li> <li>iv. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.2c)</li> <li>v. Convert a rational number to a decimal using long division. (CCSS: 7.NS.2d)</li> <li>vi. Show that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.2d)</li> </ul> c. Solve real-world and mathematical problems involving the four operations with rational numbers. <sup>8</sup> (CCSS: 7.NS.3)	<b>Inquiry Questions:</b> <ol style="list-style-type: none"> <li>1. How do operations with rational numbers compare to operations with integers?</li> <li>2. How do you know if a computational strategy is sensible?</li> <li>3. Is <math>0.\overline{9}</math> equal to one?</li> <li>4. How do you know whether a fraction can be represented as a repeating or terminating decimal?</li> </ol> <b>Relevance and Application:</b> <ol style="list-style-type: none"> <li>1. The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax.</li> <li>2. Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns.</li> <li>3. People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages).</li> </ol> <b>Nature of Mathematics:</b> <ol style="list-style-type: none"> <li>1. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to accomplish various tasks.</li> <li>2. Mathematicians make sense of problems and persevere in solving them. (MP)</li> <li>3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>4. Mathematicians look for and make use of structure. (MP)</li> </ol>

**Standard: 1. Number Sense, Properties, and Operations**

## Seventh Grade

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<sup>1</sup> For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{1/2}{1/4}$  miles per hour, equivalently 2 miles per hour. (CCSS: 7.RP.1)

<sup>2</sup> e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.2a)

<sup>3</sup> For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ . (CCSS: 7.RP.2c)

<sup>4</sup> Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS: 7.RP.3)

<sup>5</sup> For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. (CCSS: 7.NS.1a)

<sup>6</sup> Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as  $(-1)(-1) = 1$  and the rules for multiplying signed numbers. (CCSS: 7.NS.2a)

<sup>7</sup> Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If  $p$  and  $q$  are integers, then  $-(p/q) = (-p)/q = p/(-q)$ . (CCSS: 7.NS.2b)

Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.2b)

<sup>8</sup> Computations with rational numbers extend the rules for manipulating fractions to complex fractions. (CCSS: 7.NS.3)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

#### Grade Level Expectation: Sixth Grade

##### Concepts and skills students master:

1. Quantities can be expressed and compared using ratios and rates

##### Evidence Outcomes

###### Students can:

- a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.<sup>1</sup> (CCSS: 6.RP.1)
- b. Apply the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship.<sup>2</sup> (CCSS: 6.RP.2)
- c. Use ratio and rate reasoning to solve real-world and mathematical problems.<sup>3</sup> (CCSS: 6.RP.3)
  - i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. (CCSS: 6.RP.3a)
  - ii. Use tables to compare ratios. (CCSS: 6.RP.3a)
  - iii. Solve unit rate problems including those involving unit pricing and constant speed.<sup>4</sup> (CCSS: 6.RP.3b)
  - iv. Find a percent of a quantity as a rate per 100.<sup>5</sup> (CCSS: 6.RP.3c)
  - v. Solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.3c)
  - vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions (PFL)
  - vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving (PFL)
  - viii. Use ratio reasoning to convert measurement units.<sup>6</sup> (CCSS: 6.RP.3d)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. How are ratios different from fractions?
2. What is the difference between quantity and number?

###### Relevance and Application:

1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions.
2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers.
3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks.

###### Nature of Mathematics:

1. Mathematicians develop simple procedures to express complex mathematical concepts.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians reason abstractly and quantitatively. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

**Grade Level Expectation: Sixth Grade**

**Concepts and skills students master:**

- 2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency

**Evidence Outcomes**

**Students can:**

- a. Fluently divide multi-digit numbers using standard algorithms. (CCSS: 6.NS.2)
- b. Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. (CCSS: 6.NS.3)
- c. Find the greatest common factor of two whole numbers less than or equal to 100. (CCSS: 6.NS.4)
- d. Find the least common multiple of two whole numbers less than or equal to 12. (CCSS: 6.NS.4)
- e. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.<sup>7</sup> (CCSS: 6.NS.4)
- f. Interpret and model quotients of fractions through the creation of story contexts.<sup>8</sup> (CCSS: 6.NS.1)
- g. Compute quotients of fractions.<sup>9</sup> (CCSS: 6.NS.1)
- h. Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.<sup>10</sup> (CCSS: 6.NS.1)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. Why might estimation be better than an exact answer?
- 2. How do operations with fractions and decimals compare to operations with whole numbers?

**Relevance and Application:**

- 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry.

**Nature of Mathematics:**

- 1. Mathematicians envision and test strategies for solving problems.
- 2. Mathematicians model with mathematics. (MP)
- 3. Mathematicians look for and make use of structure. (MP)



## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Sixth Grade

##### Concepts and skills students master:

3. In the real number system, rational numbers have a unique location on the number line and in space

##### Evidence Outcomes

###### Students can:

- Explain why positive and negative numbers are used together to describe quantities having opposite directions or values.<sup>11</sup> (CCSS: 6.NS.5)
  - Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.5)
- Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.<sup>12</sup> (CCSS: 6.NS.6)
  - Describe a rational number as a point on the number line. (CCSS: 6.NS.6)
  - Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line. (CCSS: 6.NS.6a)
  - Identify that the opposite of the opposite of a number is the number itself.<sup>13</sup> (CCSS: 6.NS.6a)
  - Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.6b)
  - Find and position integers and other rational numbers on a horizontal or vertical number line diagram. (CCSS: 6.NS.6c)
  - Find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.6c)
- Order and find absolute value of rational numbers. (CCSS: 6.NS.7)
  - Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.<sup>14</sup> (CCSS: 6.NS.7a)
  - Write, interpret, and explain statements of order for rational numbers in real-world contexts.<sup>15</sup> (CCSS: 6.NS.7b)
  - Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.<sup>16</sup> (CCSS: 6.NS.7c)
  - Distinguish comparisons of absolute value from statements about order.<sup>17</sup> (CCSS: 6.NS.7d)
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.8)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- Why are there negative numbers?
- How do we compare and contrast numbers?
- Are there more rational numbers than integers?

###### Relevance and Application:

- Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends.
- Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down

###### Nature of Mathematics:

- Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations.
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians attend to precision. (MP)

**Standard: 1. Number Sense, Properties, and Operations**  
**Sixth Grade**

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- <sup>1</sup> For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” (CCSS: 6.RP.1)
- <sup>2</sup> For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” (CCSS: 6.RP.2)
- <sup>3</sup> e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.3)
- <sup>4</sup> For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? (CCSS: 6.RP.3b)
- <sup>5</sup> e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity. (CCSS: 6.RP.3c)
- <sup>6</sup> manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.3d)
- <sup>7</sup> For example, express  $36 + 8$  as  $4(9 + 2)$ . (CCSS: 6.NS.4)
- <sup>8</sup> For example, create a story context for  $(\frac{2}{3}) \div (\frac{3}{4})$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$  because  $\frac{3}{4}$  of  $\frac{8}{9}$  is  $\frac{2}{3}$ . (CCSS: 6.NS.1)
- <sup>9</sup> In general,  $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$ . (CCSS: 6.NS.1)
- <sup>10</sup> How much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{3}{4}$ -cup servings are in  $\frac{2}{3}$  of a cup of yogurt? How wide is a rectangular strip of land with length  $\frac{3}{4}$  mi and area  $\frac{1}{2}$  square mi? (CCSS: 6.NS.1)
- <sup>11</sup> e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). (CCSS: 6.NS.5)
- <sup>12</sup> Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. (CCSS: 6.NS.6)
- <sup>13</sup> e.g.,  $-(-3) = 3$ , and that 0 is its own opposite. (CCSS: 6.NS.6a)
- <sup>14</sup> For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right. (CCSS: 6.NS.7a)
- <sup>15</sup> For example, write  $-3\text{ }^\circ\text{C} > -7\text{ }^\circ\text{C}$  to express the fact that  $-3\text{ }^\circ\text{C}$  is warmer than  $-7\text{ }^\circ\text{C}$ . (CCSS: 6.NS.7b)
- <sup>16</sup> For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars. (CCSS: 6.NS.7c)
- <sup>17</sup> For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars. (CCSS: 6.NS.7d)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Fifth Grade

##### Concepts and skills students master:

1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

##### Evidence Outcomes

###### Students can:

- a. Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (CCSS: 5.NBT.1)
  - i. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. (CCSS: 5.NBT.2)
  - ii. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. (CCSS: 5.NBT.2)
  - iii. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.2)
- b. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.3)
  - i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.<sup>1</sup> (CCSS: 5.NBT.3a)
  - ii. Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (CCSS: 5.NBT.3b)
- c. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.4)
- d. Convert like measurement units within a given measurement system. (CCSS: 5.MD)
  - i. Convert among different-sized standard measurement units within a given measurement system.<sup>2</sup> (CCSS: 5.MD.1)
  - ii. Use measurement conversions in solving multi-step, real world problems. (CCSS: 5.MD.1)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. What is the benefit of place value system?
2. What would it mean if we did not have a place value system?
3. What is the purpose of a place value system?
4. What is the purpose of zero in a place value system?

###### Relevance and Application:

1. Place value is applied to represent a myriad of numbers using only ten symbols.

###### Nature of Mathematics:

1. Mathematicians use numbers like writers use letters to express ideas.
2. Mathematicians look closely and make use of structure by discerning patterns.
3. Mathematicians make sense of problems and persevere in solving them. (MP)
4. Mathematicians reason abstractly and quantitatively. (MP)
5. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

### Grade Level Expectation: Fifth Grade

#### Concepts and skills students master:

2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency

#### Evidence Outcomes

##### Students can:

- a. Fluently multiply multi-digit whole numbers using standard algorithms. (CCSS: 5.NBT.5)
- b. Find whole-number quotients of whole numbers.<sup>3</sup> (CCSS: 5.NBT.6)
  - i. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 5.NBT.6)
  - ii. Illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.6)
- c. Add, subtract, multiply, and divide decimals to hundredths. (CCSS: 5.NBT.7)
  - i. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 5.NBT.7)
  - ii. Relate strategies to a written method and explain the reasoning used. (CCSS: 5.NBT.7)
- d. Write and interpret numerical expressions. (CCSS: 5.OA)
  - i. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.1)
  - ii. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.<sup>4</sup> (CCSS: 5.OA.2)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

1. How are mathematical operations related?
2. What makes one strategy or algorithm better than another?

##### Relevance and Application:

1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.
2. There are many models of multiplication and division such as the area model for tiling a floor and the repeated addition to group people for games.

##### Nature of Mathematics:

1. Mathematicians envision and test strategies for solving problems.
2. Mathematicians develop simple procedures to express complex mathematical concepts.
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
4. Mathematicians model with mathematics. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

**Grade Level Expectation: Fifth Grade**

**Concepts and skills students master:**

- 3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency

**Evidence Outcomes**

**Students can:**

- a. Use equivalent fractions as a strategy to add and subtract fractions. (CCSS: 5.NF)
  - i. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.<sup>5</sup> (CCSS: 5.NF.2)
  - ii. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions<sup>6</sup> with like denominators. (CCSS: 5.NF.1)
  - iii. Solve word problems involving addition and subtraction of fractions referring to the same whole.<sup>7</sup> (CCSS: 5.NF.2)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How do operations with fractions compare to operations with whole numbers?
- 2. Why are there more fractions than whole numbers?
- 3. Is there a smallest fraction?

**Relevance and Application:**

- 1. Computational fluency with fractions is necessary for activities in daily life such as cooking and measuring for household projects and crafts.
- 2. Estimation with fractions enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, the amount of carpeting needed for a room, or fencing required for a backyard.

**Nature of Mathematics:**

- 1. Mathematicians envision and test strategies for solving problems.
- 2. Mathematicians make sense of problems and persevere in solving them. (MP)
- 3. Mathematicians reason abstractly and quantitatively. (MP)
- 4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics****Standard: 1. Number Sense, Properties, and Operations****Prepared Graduates:**

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

**Grade Level Expectation: Fifth Grade****Concepts and skills students master:**

4. The concepts of multiplication and division can be applied to multiply and divide fractions (CCSS: 5.NF)

**Evidence Outcomes****Students can:**

- a. Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). (CCSS: 5.NF.3)
- b. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.<sup>8</sup> (CCSS: 5.NF.3)
- c. Interpret the product  $(a/b) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ .<sup>9</sup> In general,  $(a/b) \times (c/d) = ac/bd$ . (CCSS: 5.NF.4a)
- d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 5.NF.4b)
  - i. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.4b)
- e. Interpret multiplication as scaling (resizing). (CCSS: 5.NF.5)
  - i. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.<sup>10</sup> (CCSS: 5.NF.5a)
  - ii. Apply the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1. (CCSS: 5.NF.5b)
- f. Solve real world problems involving multiplication of fractions and mixed numbers.<sup>11</sup> (CCSS: 5.NF.6)
- g. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.<sup>12</sup> (CCSS: 5.NF.7a)
- h. Interpret division of a whole number by a unit fraction, and compute such quotients.<sup>13</sup> (CCSS: 5.NF.7b)
- i. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.<sup>14</sup> (CCSS: 5.NF.7c)

**21<sup>st</sup> Century Skills and Readiness Competencies****Inquiry Questions:**

1. Do adding and multiplying always result in an increase? Why?
2. Do subtracting and dividing always result in a decrease? Why?
3. How do operations with fractional numbers compare to operations with whole numbers?

**Relevance and Application:**

1. Rational numbers are used extensively in measurement tasks such as home remodeling, clothes alteration, graphic design, and engineering.
2. Situations from daily life can be modeled using operations with fractions, decimals, and percents such as determining the quantity of paint to buy or the number of pizzas to order for a large group.
3. Rational numbers are used to represent data and probability such as getting a certain color of gumball out of a machine, the probability that a batter will hit a home run, or the percent of a mountain covered in forest.

**Nature of Mathematics:**

1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians model with mathematics. (MP)
4. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Standard: 1. Number Sense, Properties, and Operations**  
**Fifth Grade**

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<sup>1</sup> e.g.,  $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ . (CCSS: 5.NBT.3a)

<sup>2</sup> e.g., convert 5 cm to 0.05 m. (CCSS: 5.MD.1)

<sup>3</sup> with up to four-digit dividends and two-digit divisors. (CCSS: 5.NBT.6)

<sup>4</sup> For example, express the calculation "add 8 and 7, then multiply by 2" as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product. (CCSS: 5.OA.2)

<sup>5</sup> For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ . (CCSS: 5.NF.2)

<sup>6</sup> in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ . (In general,  $a/b + c/d = (ad + bc)/bd$ .) (CCSS: 5.NF.1)

<sup>7</sup> including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.2)

<sup>8</sup> e.g., by using visual fraction models or equations to represent the problem. For example, interpret  $3/4$  as the result of dividing 3 by 4, noting that  $3/4$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $3/4$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS: 5.NF.3)

<sup>9</sup> For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (CCSS: 5.NF.4a)

<sup>10</sup> Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. (CCSS: 5.NF.5b)

Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number (CCSS: 5.NF.5b)

<sup>11</sup> e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.6)

<sup>12</sup> For example, create a story context for  $(1/3) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(1/3) \div 4 = 1/12$  because  $(1/12) \times 4 = 1/3$ . (CCSS: 5.NF.7a)

<sup>13</sup> For example, create a story context for  $4 \div (1/5)$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (1/5) = 20$  because  $20 \times (1/5) = 4$ . (CCSS: 5.NF.7b)

<sup>14</sup> e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $1/3$ -cup servings are in 2 cups of raisins? (CCSS: 5.NF.7c)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

##### Evidence Outcomes

###### Students can:

- a. Generalize place value understanding for multi-digit whole numbers (CCSS: 4.NBT)
  - i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)
  - ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. (CCSS: 4.NBT.2)
  - iii. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (CCSS: 4.NBT.2)
  - iv. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)
- b. Use decimal notation to express fractions, and compare decimal fractions (CCSS: 4.NF)
  - i. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.<sup>1</sup> (CCSS: 4.NF.5)
  - ii. Use decimal notation for fractions with denominators 10 or 100.<sup>2</sup> (CCSS: 4.NF.6)
  - iii. Compare two decimals to hundredths by reasoning about their size.<sup>3</sup> (CCSS: 4.NF.7)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. Why isn't there a "oneths" place in decimal fractions?
2. How can a number with greater decimal digits be less than one with fewer decimal digits?
3. Is there a decimal closest to one? Why?

###### Relevance and Application:

1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated.
2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events.

###### Nature of Mathematics:

1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians look for and make use of structure. (MP)



## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

#### Grade Level Expectation: Fourth Grade

#### Concepts and skills students master:

2. Different models and representations can be used to compare fractional parts

#### Evidence Outcomes

##### Students can:

- Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF)
  - Explain equivalence of fractions using drawings and models.<sup>4</sup>
  - Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1)
  - Compare two fractions with different numerators and different denominators,<sup>5</sup> and justify the conclusions.<sup>6</sup> (CCSS: 4.NF.2)
- Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)
  - Apply previous understandings of addition and subtraction to add and subtract fractions.<sup>7</sup>
    - Compose and decompose fractions as sums and differences of fractions with the same denominator in more than one way and justify with visual models.
    - Add and subtract mixed numbers with like denominators.<sup>8</sup> (CCSS: 4.NF.3c)
    - Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.<sup>9</sup> (CCSS: 4.NF.3d)
  - Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.4)
    - Express a fraction  $a/b$  as a multiple of  $1/b$ .<sup>10</sup> (CCSS: 4.NF.4a)
    - Use a visual fraction model to express  $a/b$  as a multiple of  $1/b$ , and apply to multiplication of whole number by a fraction.<sup>11</sup> (CCSS: 4.NF.4b)
    - Solve word problems involving multiplication of a fraction by a whole number.<sup>12</sup> (CCSS: 4.NF.4c)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

- How can different fractions represent the same quantity?
- How are fractions used as models?
- Why are fractions so useful?
- What would the world be like without fractions?

##### Relevance and Application:

- Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies.
- Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal.
- Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon.

##### Nature of Mathematics:

- Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Are fluent with basic numerical, symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency

##### Evidence Outcomes

###### Students can:

- Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT)
  - Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4)
  - Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5)
  - Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6)
  - Illustrate and explain multiplication and division calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.6)
- Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)
  - Interpret a multiplication equation as a comparison.<sup>13</sup> (CCSS: 4.OA.1)
  - Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.1)
  - Multiply or divide to solve word problems involving multiplicative comparison.<sup>14</sup> (CCSS: 4.OA.2)
  - Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3)
  - Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)
  - Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)
  - Using the four operations analyze the relationship between choice and opportunity cost (PFL)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- Is it possible to make multiplication and division of large numbers easy?
- What do remainders mean and how are they used?
- When is the "correct" answer not the most useful answer?

###### Relevance and Application:

- Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.

###### Nature of Mathematics:

- Mathematicians envision and test strategies for solving problems.
- Mathematicians develop simple procedures to express complex mathematical concepts.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians look for and express regularity in repeated reasoning. (MP)

## Standard: 1. Number Sense, Properties, and Operations Fourth Grade

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<sup>1</sup> For example, express  $3/10$  as  $30/100$ , and add  $3/10 + 4/100 = 34/100$ . (CCSS: 4.NF.6)

<sup>2</sup> For example, rewrite  $0.62$  as  $62/100$ ; describe a length as  $0.62$  meters; locate  $0.62$  on a number line diagram. (CCSS: 4.NF.6)

<sup>3</sup> Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.7)

<sup>4</sup> Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (CCSS: 4.NF.1)

<sup>5</sup> e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , (CCSS: 4.NF.2)

<sup>6</sup> e.g., by using a visual fraction model. (CCSS: 4.NF.2)

<sup>7</sup> Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ . (CCSS: 4.NF.3)

Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.3a)

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:*  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ . (CCSS: 4.NF.3b)

<sup>8</sup> e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.3c)

<sup>9</sup> e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.3d)

<sup>10</sup> For example, use a visual fraction model to represent  $5/4$  as the product  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ . (CCSS: 4.NF.4a)

<sup>11</sup> For example,  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .) (CCSS: 4.NF.4b)

<sup>12</sup> e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.4c)

<sup>13</sup> e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)

<sup>14</sup> e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

1. The whole number system describes place value relationships and forms the foundation for efficient algorithms

**Evidence Outcomes**

**Students can:**

- a. Use place value and properties of operations to perform multi-digit arithmetic. (CCSS: 3.NBT)
  - i. Use place value to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.1)
  - ii. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.2)
  - iii. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations.<sup>1</sup> (CCSS: 3.NBT.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. How do patterns in our place value system assist in comparing whole numbers?
2. How might the most commonly used number system be different if humans had twenty fingers instead of ten?

**Relevance and Application:**

1. Knowledge and use of place value for large numbers provides context for distance in outer space, prehistoric timelines, and ants in a colony.
2. The building and taking apart of numbers provide a deep understanding of the base 10 number system.

**Nature of Mathematics:**

1. Mathematicians use numbers like writers use letters to express ideas.
2. Mathematicians look for and make use of structure. (MP)
3. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

2. Parts of a whole can be modeled and represented in different ways

**Evidence Outcomes**

**Students can:**

- a. Develop understanding of fractions as numbers. (CCSS: 3.NF)
  - i. Describe a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; describe a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ . (CCSS: 3.NF.1)
  - ii. Describe a fraction as a number on the number line; represent fractions on a number line diagram.<sup>2</sup> (CCSS: 3.NF.2)
  - iii. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.3)
    - 1. Identify two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.3a)
    - 2. Identify and generate simple equivalent fractions. Explain<sup>3</sup> why the fractions are equivalent.<sup>4</sup> (CCSS: 3.NF.3b)
    - 3. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.<sup>5</sup> (CCSS: 3.NF.3c)
    - 4. Compare two fractions with the same numerator or the same denominator by reasoning about their size. (CCSS: 3.NF.3d)
    - 5. Explain why comparisons are valid only when the two fractions refer to the same whole. (CCSS: 3.NF.3d)
    - 6. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions.<sup>6</sup> (CCSS: 3.NF.3d)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How many ways can a whole number be represented?
- 2. How can a fraction be represented in different, equivalent forms?
- 3. How do we show part of unit?

**Relevance and Application:**

- 1. Fractions are used to share fairly with friends and family such as sharing an apple with a sibling, and splitting the cost of lunch.
- 2. Equivalent fractions demonstrate equal quantities even when they are presented differently such as knowing that  $1/2$  of a box of crayons is the same as  $2/4$ , or that  $2/6$  of the class is the same as  $1/3$ .

**Nature of Mathematics:**

- 1. Mathematicians use visual models to solve problems.
- 2. Mathematicians make sense of problems and persevere in solving them. (MP)
- 3. Mathematicians reason abstractly and quantitatively. (MP)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

### Grade Level Expectation: Third Grade

#### Concepts and skills students master:

#### 3. Multiplication and division are inverse operations and can be modeled in a variety of ways

#### Evidence Outcomes

##### Students can:

- Represent and solve problems involving multiplication and division. (CCSS: 3.OA)
  - Interpret products of whole numbers.<sup>7</sup> (CCSS: 3.OA.1)
  - Interpret whole-number quotients of whole numbers.<sup>8</sup> (CCSS: 3.OA.2)
  - Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.<sup>9</sup> (CCSS: 3.OA.3)
  - Determine the unknown whole number in a multiplication or division equation relating three whole numbers.<sup>10</sup> (CCSS: 3.OA.4)
  - Model strategies to achieve a personal financial goal using arithmetic operations (PFL)
- Apply properties of multiplication and the relationship between multiplication and division. (CCSS: 3.OA)
  - Apply properties of operations as strategies to multiply and divide.<sup>11</sup> (CCSS: 3.OA.5)
  - Interpret division as an unknown-factor problem.<sup>12</sup> (CCSS: 3.OA.6)
- Multiply and divide within 100. (CCSS: 3.OA)
  - Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division<sup>13</sup> or properties of operations. (CCSS: 3.OA.7)
  - Recall from memory all products of two one-digit numbers. (CCSS: 3.OA.7)
- Solve problems involving the four operations, and identify and explain patterns in arithmetic. (CCSS: 3.OA)
  - Solve two-step word problems using the four operations. (CCSS: 3.OA.8)
  - Represent two-step word problems using equations with a letter standing for the unknown quantity. (CCSS: 3.OA.8)
  - Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 3.OA.8)
  - Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.<sup>14</sup> (CCSS: 3.OA.9)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

- How are multiplication and division related?
- How can you use a multiplication or division fact to find a related fact?
- Why was multiplication invented? Why not just add?
- Why was division invented? Why not just subtract?

##### Relevance and Application:

- Many situations in daily life can be modeled with multiplication and division such as how many tables to set up for a party, how much food to purchase for the family, or how many teams can be created.
- Use of multiplication and division helps to make decisions about spending allowance or gifts of money such as how many weeks of saving an allowance of \$5 per week to buy a soccer ball that costs \$32?.

##### Nature of Mathematics:

- Mathematicians often learn concepts on a smaller scale before applying them to a larger situation.
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians model with mathematics. (MP)
- Mathematicians look for and make use of structure. (MP)

**Standard: 1. Number Sense, Properties, and Operations**  
**Third Grade**

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<sup>1</sup> e.g.,  $9 \times 80$ ,  $5 \times 60$ . (CCSS: 3.NBT.3)

<sup>2</sup> Represent a fraction  $1/b$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line. (CCSS: 3.NF.2a)

Represent a fraction  $a/b$  on a number line diagram by marking off  $a$  lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line. (CCSS: 3.NF.2b)

<sup>3</sup> e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ . (CCSS: 3.NF.3b)

<sup>4</sup> e.g., by using a visual fraction model. (CCSS: 3.NF.3b)

<sup>5</sup> Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at the same point of a number line diagram. (CCSS: 3.NF.3c)

<sup>6</sup> e.g., by using a visual fraction model. (CCSS: 3.NF.3d)

<sup>7</sup> e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. (CCSS: 3.OA.1)

For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ . (CCSS: 3.OA.1)

<sup>8</sup> e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. (CCSS: 3.OA.2)

For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ . (CCSS: 3.OA.2)

<sup>9</sup> e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 3.OA.3)

<sup>10</sup> For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \square \div 3$ ,  $6 \times 6 = ?$ . (CCSS: 3.OA.4)

<sup>11</sup> Examples: If  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known. (Commutative property of multiplication.)  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$ , then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$ , then  $3 \times 10 = 30$ . (Associative property of multiplication.) Knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . (Distributive property.) (CCSS: 3.OA.5)

<sup>12</sup> For example, find  $32 \div 8$  by finding the number that makes 32 when multiplied by 8. (CCSS: 3.OA.6)

<sup>13</sup> e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ . (CCSS: 3.OA.7)

<sup>14</sup> For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. (CCSS: 3.OA.9)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Second Grade

##### Concepts and skills students master:

1. The whole number system describes place value relationships through 1,000 and forms the foundation for efficient algorithms

##### Evidence Outcomes

###### Students can:

- a. Use place value to read, write, count, compare, and represent numbers. (CCSS: 2.NBT)
  - i. Represent the digits of a three-digit number as hundreds, tens, and ones.<sup>1</sup> (CCSS: 2.NBT.1)
  - ii. Count within 1000. (CCSS: 2.NBT.2)
  - iii. Skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.2)
  - iv. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.3)
  - v. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons. (CCSS: 2.NBT.4)
- b. Use place value understanding and properties of operations to add and subtract. (CCSS: 2.NBT)
  - i. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.5)
  - ii. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.6)
  - iii. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.<sup>2</sup> (CCSS: 2.NBT.7)
  - iv. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.8)
  - v. Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS: 2.NBT.9)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. How big is 1,000?
2. How does the position of a digit in a number affect its value?

###### Relevance and Application:

1. The ability to read and write numbers allows communication about quantities such as the cost of items, number of students in a school, or number of people in a theatre.
2. Place value allows people to represent large quantities. For example, 725 can be thought of as  $700 + 20 + 5$ .

###### Nature of Mathematics:

1. Mathematicians use place value to represent many numbers with only ten digits.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians look for and make use of structure. (MP)
4. Mathematicians look for and express regularity in repeated reasoning. (MP)



## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

#### Grade Level Expectation: Second Grade

##### Concepts and skills students master:

2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

##### Evidence Outcomes

###### Students can:

- Represent and solve problems involving addition and subtraction. (CCSS: 2.OA)
  - Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.<sup>3</sup> (CCSS: 2.OA.1)
  - Apply addition and subtraction concepts to financial decision-making (PFL)
- Fluently add and subtract within 20 using mental strategies. (CCSS: 2.OA.2)
- Know from memory all sums of two one-digit numbers. (CCSS: 2.OA.2)
- Use equal groups of objects to gain foundations for multiplication. (CCSS: 2.OA)
  - Determine whether a group of objects (up to 20) has an odd or even number of members.<sup>4</sup> (CCSS: 2.OA.3)
  - Write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.3)
  - Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.4)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. What are the ways numbers can be broken apart and put back together?
2. What could be a result of not using pennies (taking them out of circulation)?

###### Relevance and Application:

1. Addition is used to find the total number of objects such as total number of animals in a zoo, total number of students in first and second grade.
2. Subtraction is used to solve problems such as how many objects are left in a set after taking some away, or how much longer one line is than another.
3. The understanding of the value of a collection of coins helps to determine how many coins are used for a purchase or checking that the amount of change is correct.

###### Nature of Mathematics:

1. Mathematicians use visual models to understand addition and subtraction.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians reason abstractly and quantitatively. (MP)
4. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Standard: 1. Number Sense, Properties, and Operations**  
**Second Grade**

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<sup>1</sup> e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.1)

100 can be thought of as a bundle of ten tens — called a “hundred.” (CCSS: 2.NBT.1a)

The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.1b)

<sup>2</sup> Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.7)

<sup>3</sup> e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.OA.1)

<sup>4</sup> e.g., by pairing objects or counting them by 2s. (CCSS: 2.OA.3)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: First Grade

##### Concepts and skills students master:

1. The whole number system describes place value relationships within and beyond 100 and forms the foundation for efficient algorithms

##### Evidence Outcomes

###### Students can:

- a. Count to 120 (CCSS: 1.NBT.1)
  - i. Count starting at any number less than 120. (CCSS: 1.NBT.1)
  - ii. Within 120, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.1)
- b. Represent and use the digits of a two-digit number. (CCSS: 1.NBT.2)
  - i. Represent the digits of a two-digit number as tens and ones.<sup>1</sup> (CCSS: 1.NBT.2)
  - ii. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols  $>$ ,  $=$ , and  $<$ . (CCSS: 1.NBT.3)
  - iii. Compare two sets of objects, including pennies, up to at least 25 using language such as "three more or three fewer" (PFL)
- c. Use place value and properties of operations to add and subtract. (CCSS: 1.NBT)
  - i. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten, using concrete models or drawings, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.4)
  - ii. Identify coins and find the value of a collection of two coins (PFL)
  - iii. Mentally find 10 more or 10 less than any two-digit number, without counting; explain the reasoning used. (CCSS: 1.NBT.5)
  - iv. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 1.NBT.6)
  - v. Relate addition and subtraction strategies to a written method and explain the reasoning used. (CCSS: 1.NBT.4 and 1.NBT.6)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. Can numbers always be related to tens?
2. Why not always count by one?
3. Why was a place value system developed?
4. How does a position of a digit affect its value?
5. How big is 100?

###### Relevance and Application:

1. The comparison of numbers helps to communicate and to make sense of the world. (For example, if someone has two more dollars than another, gets four more points than another, or takes out three fewer forks than needed.)

###### Nature of Mathematics:

1. Mathematics involves visualization and representation of ideas.
2. Numbers are used to count and order both real and imaginary objects.
3. Mathematicians reason abstractly and quantitatively. (MP)
4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: First Grade**

**Concepts and skills students master:**

2. Number relationships can be used to solve addition and subtraction problems

**Evidence Outcomes**

**Students can:**

- a. Represent and solve problems involving addition and subtraction. (CCSS: 1.OA)
  - i. Use addition and subtraction within 20 to solve word problems.<sup>2</sup> (CCSS: 1.OA.1)
  - ii. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.<sup>3</sup> (CCSS: 1.OA.2)
- b. Apply properties of operations and the relationship between addition and subtraction. (CCSS: 1.OA)
  - i. Apply properties of operations as strategies to add and subtract.<sup>4</sup> (CCSS: 1.OA.3)
  - ii. Relate subtraction to unknown-addend problem.<sup>5</sup> (CCSS: 1.OA.4)
- c. Add and subtract within 20. (CCSS: 1.OA)
  - i. Relate counting to addition and subtraction.<sup>6</sup> (CCSS: 1.OA.5)
  - ii. Add and subtract within 20 using multiple strategies.<sup>7</sup> (CCSS: 1.OA.6)
  - iii. Demonstrate fluency for addition and subtraction within 10. (CCSS: 1.OA.6)
- d. Use addition and subtraction equations to show number relationships. (CCSS: 1.OA)
  - i. Use the equal sign to demonstrate equality in number relationships.<sup>8</sup> (CCSS: 1.OA.7)
  - ii. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.<sup>9</sup> (CCSS: 1.OA.8)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. What is addition and how is it used?
- 2. What is subtraction and how is it used?
- 3. How are addition and subtraction related?

**Relevance and Application:**

- 1. Addition and subtraction are used to model real-world situations such as computing saving or spending, finding the number of days until a special day, or determining an amount needed to earn a reward.
- 2. Fluency with addition and subtraction facts helps to quickly find answers to important questions.

**Nature of Mathematics:**

- 1. Mathematicians use addition and subtraction to take numbers apart and put them back together in order to understand number relationships.
- 2. Mathematicians make sense of problems and persevere in solving them. (MP)
- 3. Mathematicians look for and make use of structure. (MP)

## Standard: 1. Number Sense, Properties, and Operations First Grade

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<sup>1</sup> 10 can be thought of as a bundle of ten ones — called a “ten.” (CCSS: 1.NBT.2a)

The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: 1.NBT.2b)

The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS: 1.NBT.2c)

<sup>2</sup> involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.1)

<sup>3</sup> e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.2)

<sup>4</sup> Examples: If  $8 + 3 = 11$  is known, then  $3 + 8 = 11$  is also known. (Commutative property of addition.) To add  $2 + 6 + 4$ , the second two numbers can be added to make a ten, so  $2 + 6 + 4 = 2 + 10 = 12$ . (Associative property of addition.). (CCSS: 1.OA.3)

<sup>5</sup> For example, subtract  $10 - 8$  by finding the number that makes 10 when added to 8. (CCSS: 1.OA.4)

<sup>6</sup> e.g., by counting on 2 to add 2. (CCSS: 1.OA.5)

<sup>7</sup> Use strategies such as counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ ); decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ ); using the relationship between addition and subtraction (e.g., knowing that  $8 + 4 = 12$ , one knows  $12 - 8 = 4$ ); and creating equivalent but easier or known sums (e.g., adding  $6 + 7$  by creating the known equivalent  $6 + 6 + 1 = 12 + 1 = 13$ ). (CCSS: 1.OA.6)

<sup>8</sup> Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?  $6 = 6$ ,  $7 = 8 - 1$ ,  $5 + 2 = 2 + 5$ ,  $4 + 1 = 5 + 2$ .* (CCSS: 1.OA.7)

<sup>9</sup> For example, determine the unknown number that makes the equation true in each of the equations  $8 + ? = 11$ ,  $5 = \square - 3$ ,  $6 + 6 = \square$  (CCSS: 1.OA.8)

## Content Area: Mathematics

### Standard: 1. Number Sense, Properties, and Operations

#### Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

#### Grade Level Expectation: Kindergarten

##### Concepts and skills students master:

1. Whole numbers can be used to name, count, represent, and order quantity

##### Evidence Outcomes

###### Students can:

- a. Use number names and the count sequence. (CCSS: K.CC)
  - i. Count to 100 by ones and by tens. (CCSS: K.CC.1)
  - ii. Count forward beginning from a given number within the known sequence.<sup>1</sup> (CCSS: K.CC.2)
  - iii. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.<sup>2</sup> (CCSS: K.CC.3)
- b. Count to determine the number of objects. (CCSS: K.CC)
  - i. Apply the relationship between numbers and quantities and connect counting to cardinality.<sup>3</sup> (CCSS: K.CC.4)
  - ii. Count and represent objects to 20.<sup>4</sup> (CCSS: K.CC.5)
- c. Compare and instantly recognize numbers. (CCSS: K.CC)
  - i. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.<sup>5</sup> (CCSS: K.CC.6)
  - ii. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.7)
  - iii. Identify small groups of objects fewer than five without counting

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. Why do we count things?
2. Is there a wrong way to count? Why?
3. How do you know when you have more or less?
4. What does it mean to be second and how is it different than two?

###### Relevance and Application:

1. Counting is used constantly in everyday life such as counting plates for the dinner table, people on a team, pets in the home, or trees in a yard.
2. Numerals are used to represent quantities.
3. People use numbers to communicate with others such as two more forks for the dinner table, one less sister than my friend, or six more dollars for a new toy.

###### Nature of Mathematics:

1. Mathematics involves visualization and representation of ideas.
2. Numbers are used to count and order both real and imaginary objects.
3. Mathematicians attend to precision. (MP)
4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: Kindergarten**

**Concepts and skills students master:**

**2. Composing and decomposing quantity forms the foundation for addition and subtraction**

**Evidence Outcomes**

**Students can:**

- a. Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings. (CCSS: K.OA)
  - i. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds,<sup>6</sup> acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.1)
  - ii. Solve addition and subtraction word problems, and add and subtract within 10.<sup>7</sup> (CCSS: K.OA.2)
  - iii. Decompose numbers less than or equal to 10 into pairs in more than one way.<sup>8</sup> (CCSS: K.OA.3)
  - iv. For any number from 1 to 9, find the number that makes 10 when added to the given number.<sup>9</sup> (CCSS: K.OA.4)
  - v. Use objects including coins and drawings to model addition and subtraction problems to 10 (PFL)
- b. Fluently add and subtract within 5. (CCSS: K.OA.5)
- c. Compose and decompose numbers 11–19 to gain foundations for place value using objects and drawings.<sup>10</sup> (CCSS: K.NBT)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. What happens when two quantities are combined?
- 2. What happens when a set of objects is separated into different sets?

**Relevance and Application:**

- 1. People combine quantities to find a total such as number of boys and girls in a classroom or coins for a purchase.
- 2. People use subtraction to find what is left over such as coins left after a purchase, number of toys left after giving some away.

**Nature of Mathematics:**

- 1. Mathematicians create models of problems that reveal relationships and meaning.
- 2. Mathematics involves the creative use of imagination.
- 3. Mathematicians reason abstractly and quantitatively. (MP)
- 4. Mathematicians model with mathematics. (MP)

## **Standard: 1. Number Sense, Properties, and Operations Kindergarten**

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<sup>1</sup> instead of having to begin at 1. (CCSS: K.CC.2)

<sup>2</sup> with 0 representing a count of no objects. (CCSS: K.CC.3)

<sup>3</sup> When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.4a)

Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.4b)

Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.4c)

<sup>4</sup> Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration. (CCSS: K.CC.5)

Given a number from 1–20, count out that many objects. (CCSS: K.CC.5)

<sup>5</sup> e.g., by using matching and counting strategies. (CCSS: K.CC.6)

<sup>6</sup> e.g., claps. (CCSS: K.OA.1)

<sup>7</sup> e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.2)

<sup>8</sup> e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ). (CCSS: K.OA.3)

<sup>9</sup> e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.4)

<sup>10</sup> Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.1)



**Content Area: Mathematics**

**Standard: 1. Number Sense, Properties, and Operations**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Preschool**

**Concepts and skills students master:**

1. Quantities can be represented and counted

**Evidence Outcomes**

**Students can:**

- a. Count and represent objects including coins to 10 (PFL)
- b. Match a quantity with a numeral

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What do numbers tell us?
2. Is there a biggest number?

**Relevance and Application:**

1. Counting helps people to determine how many such as how big a family is, how many pets there are, such as how many members in one's family, how many mice on the picture book page, how many counting bears in the cup.
2. People sort things to make sense of sets of things such as sorting pencils, toys, or clothes.

**Nature of Mathematics:**

1. Numbers are used to count and order objects.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians attend to precision. (MP)

## 2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

### Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

#### **Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:**

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Content Area: Mathematics****Standard: 2. Patterns, Functions, and Algebraic Structures****Prepared Graduates:**

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

**Grade Level Expectation: High School****Concepts and skills students master:**

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables

**Evidence Outcomes****Students can:**

- a. Formulate the concept of a function and use function notation. (CCSS: F-IF)
  - i. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.<sup>1</sup> (CCSS: F-IF.1)
  - ii. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: F-IF.2)
  - iii. Demonstrate that sequences are functions,<sup>2</sup> sometimes defined recursively, whose domain is a subset of the integers. (CCSS: F-IF.3)
- b. Interpret functions that arise in applications in terms of the context. (CCSS: F-IF)
  - i. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features<sup>3</sup> given a verbal description of the relationship. \* (CCSS: F-IF.4)
  - ii. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.<sup>4</sup> \* (CCSS: F-IF.5)
  - iii. Calculate and interpret the average rate of change<sup>5</sup> of a function over a specified interval. Estimate the rate of change from a graph.\* (CCSS: F-IF.6)
- c. Analyze functions using different representations. (CCSS: F-IF)
  - i. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. \* (CCSS: F-IF.7)
  - ii. Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: F-IF.7a)
  - iii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: F-IF.7b)
  - iv. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: F-IF.7c)
  - v. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: F-IF.7e)
  - vi. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: F-IF.8)
    1. Use the process of factoring and completing the square in a quadratic

**21<sup>st</sup> Century Skills and Readiness Competencies****Inquiry Questions:**

1. Why are relations and functions represented in multiple ways?
2. How can a table, graph, and function notation be used to explain how one function family is different from and/or similar to another?
3. What is an inverse?
4. How is "inverse function" most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations?
5. How are patterns and functions similar and different?
6. How could you visualize a function with four variables, such as  $x^2 + y^2 + z^2 + w^2 = 1$ ?
7. Why couldn't people build skyscrapers without using functions?
8. How do symbolic transformations affect an equation, inequality, or expression?

**Relevance and Application:**

1. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments. (PFL)
2. Comprehension of rate of change of a function is important preparation for the study of calculus.
3. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot.
4. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function.

<p>function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: F-IF.8a)</p> <ol style="list-style-type: none"> <li>2. Use the properties of exponents to interpret expressions for exponential functions.<sup>6</sup> (CCSS: F-IF.8b)</li> <li>3. Compare properties of two functions each represented in a different way<sup>7</sup> (algebraically, graphically, numerically in tables, or by verbal descriptions). (CCSS: F-IF.9)</li> </ol> <p>d. Build a function that models a relationship between two quantities. (CCSS: F-BF)</p> <ol style="list-style-type: none"> <li>i. Write a function that describes a relationship between two quantities.* (CCSS: F-BF.1) <ol style="list-style-type: none"> <li>1. Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: F-BF.1a)</li> <li>2. Combine standard function types using arithmetic operations.<sup>8</sup> (CCSS: F-BF.1b)</li> </ol> </li> <li>ii. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* (CCSS: F-BF.2)</li> </ol> <p>e. Build new functions from existing functions. (CCSS: F-BF)</p> <ol style="list-style-type: none"> <li>i. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math>,<sup>9</sup> and find the value of <math>k</math> given the graphs.<sup>10</sup> (CCSS: F-BF.3)</li> <li>ii. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</li> <li>iii. Find inverse functions.<sup>11</sup> (CCSS: F-BF.4)</li> </ol> <p>f. Extend the domain of trigonometric functions using the unit circle. (CCSS: F-TF)</p> <ol style="list-style-type: none"> <li>i. Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: F-TF.1)</li> <li>ii. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: F-TF.2)</li> </ol> <p><i>*Indicates a part of the standard connected to the mathematical practice of Modeling</i></p>	<ol style="list-style-type: none"> <li>5. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans.</li> <li>6. Comprehension of slope, intercepts, and common forms of linear equations allows easy retrieval of information from linear models such as rate of growth or decrease, an initial charge for services, speed of an object, or the beginning balance of an account.</li> <li>7. Understanding sequences is important preparation for calculus. Sequences can be used to represent functions including <math>e^x</math>, <math>e^{x^2}</math>, <math>\sin x</math>, and <math>\cos x</math>.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematicians use multiple representations of functions to explore the properties of functions and the properties of families of functions.</li> <li>2. Mathematicians model with mathematics. (MP)</li> <li>3. Mathematicians use appropriate tools strategically. (MP)</li> <li>4. Mathematicians look for and make use of structure. (MP)</li> </ol>
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**Content Area: Mathematics**  
**Standard: 2. Patterns, Functions, and Algebraic Structures**

**Prepared Graduates:**  
 ➤ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Grade Level Expectation: High School**

**Concepts and skills students master:**  
 2. Quantitative relationships in the real world can be modeled and solved using functions

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <p>a. Construct and compare linear, quadratic, and exponential models and solve problems. (CCSS: F-LE)</p> <p>i. Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: F-LE.1)</p> <ol style="list-style-type: none"> <li>1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: F-LE.1a)</li> <li>2. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: F-LE.1b)</li> <li>3. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: F-LE.1c)</li> </ol> <p>ii. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.<sup>12</sup> (CCSS: F-LE.2)</p> <p>iii. Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: F-LE.3)</p> <p>iv. For exponential models, express as a logarithm the solution to <math>ab^{ct} = d</math> where <math>a</math>, <math>c</math>, and <math>d</math> are numbers and the base <math>b</math> is 2, 10, or <math>e</math>; evaluate the logarithm using technology. (CCSS: F-LE.4)</p> <p>b. Interpret expressions for function in terms of the situation they model. (CCSS: F-LE)</p> <p>i. Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: F-LE.5)</p> <p>c. Model periodic phenomena with trigonometric functions. (CCSS: F-TF)</p> <p>i. Choose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. * (CCSS: F-TF.5)</p> <p>d. Model personal financial situations</p> <p>i. Analyze* the impact of interest rates on a personal financial plan (PFL)</p> <p>ii. Evaluate* the costs and benefits of credit (PFL)</p> <p>iii. Analyze various lending sources, services, and financial institutions (PFL)</p> <p><i>*Indicates a part of the standard connected to the mathematical practice of Modeling.</i></p>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>1. Why do we classify functions?</li> <li>2. What phenomena can be modeled with particular functions?</li> <li>3. Which financial applications can be modeled with exponential functions? Linear functions? (PFL)</li> <li>4. What elementary function or functions best represent a given scatter plot of two-variable data?</li> <li>5. How much would today's purchase cost tomorrow? (PFL)</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>1. The understanding of the qualitative behavior of functions allows interpretation of the qualitative behavior of systems modeled by functions such as time-distance, population growth, decay, heat transfer, and temperature of the ocean versus depth.</li> <li>2. The knowledge of how functions model real-world phenomena allows exploration and improved understanding of complex systems such as how population growth may affect the environment, how interest rates or inflation affect a personal budget, how stopping distance is related to reaction time and velocity, and how volume and temperature of a gas are related.</li> <li>3. Biologists use polynomial curves to model the shapes of jaw bone fossils. They analyze the polynomials to find potential evolutionary relationships among the species.</li> <li>4. Physicists use basic linear and quadratic functions to model the motion of projectiles.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematicians use their knowledge of functions to create accurate models of complex systems.</li> <li>2. Mathematicians use models to better understand systems and make predictions about future systemic behavior.</li> <li>3. Mathematicians reason abstractly and quantitatively. (MP)</li> <li>4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>5. Mathematicians model with mathematics. (MP)</li> </ol>

**Content Area: Mathematics**  
**Standard: 2. Patterns, Functions, and Algebraic Structures**

<b>Prepared Graduates:</b>	
➤ Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations	
<b>Grade Level Expectation: High School</b>	
<b>Concepts and skills students master:</b>	
3. Expressions can be represented in multiple, equivalent forms	
<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <p>a. Interpret the structure of expressions.(CCSS: A-SSE)</p> <p>i. Interpret expressions that represent a quantity in terms of its context.* (CCSS: A-SSE.1)</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: A-SSE.1a)</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity.<sup>13</sup> (CCSS: A-SSE.1b)</li> </ol> <p>ii. Use the structure of an expression to identify ways to rewrite it.<sup>14</sup> (CCSS: A-SSE.2)</p> <p>b. Write expressions in equivalent forms to solve problems. (CCSS: A-SSE)</p> <p>i. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (CCSS: A-SSE.3)</p> <ol style="list-style-type: none"> <li>Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: A-SSE.3a)</li> <li>Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: A-SSE.3b)</li> <li>Use the properties of exponents to transform expressions for exponential functions.<sup>15</sup> (CCSS: A-SSE.3c)</li> </ol> <p>ii. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.<sup>16*</sup> (CCSS: A-SSE.4)</p> <p>c. Perform arithmetic operations on polynomials. (CCSS: A-APR)</p> <p>i. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: A-APR.1)</p> <p>d. Understand the relationship between zeros and factors of polynomials. (CCSS: A-APR)</p> <p>i. State and apply the Remainder Theorem.<sup>17</sup> (CCSS: A-APR.2)</p> <p>ii. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: A-APR.3)</p> <p>e. Use polynomial identities to solve problems. (CCSS: A-APR)</p> <p>i. Prove polynomial identities<sup>18</sup> and use them to describe numerical relationships. (CCSS: A-APR.4)</p> <p>f. Rewrite rational expressions. (CCSS: A-APR)</p> <p>g. Rewrite simple rational expressions in different forms.<sup>19</sup> (CCSS: A-APR.6)</p>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>When is it appropriate to simplify expressions?</li> <li>The ancient Greeks multiplied binomials and found the roots of quadratic equations without algebraic notation. How can this be done?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>The simplification of algebraic expressions and solving equations are tools used to solve problems in science. Scientists represent relationships between variables by developing a formula and using values obtained from experimental measurements and algebraic manipulation to determine values of quantities that are difficult or impossible to measure directly such as acceleration due to gravity, speed of light, and mass of the earth.</li> <li>The manipulation of expressions and solving formulas are techniques used to solve problems in geometry such as finding the area of a circle, determining the volume of a sphere, calculating the surface area of a prism, and applying the Pythagorean Theorem.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions.</li> <li>Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>Mathematicians model with mathematics. (MP)</li> <li>Mathematicians look for and express regularity in repeated reasoning. (MP)</li> </ol>
*Indicates a part of the standard connected to the mathematical practice of Modeling	

**Content Area: Mathematics**

## Standard: 2. Patterns, Functions, and Algebraic Structures

<p><b>Prepared Graduates:</b></p> <ul style="list-style-type: none"> <li>➤ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency</li> </ul>	
<p><b>Grade Level Expectation: High School</b></p>	
<p><b>Concepts and skills students master:</b></p> <p>4. Solutions to equations, inequalities and systems of equations are found using a variety of tools</p>	
<p><b>Evidence Outcomes</b></p>	<p><b>21<sup>st</sup> Century Skills and Readiness Competencies</b></p>
<p><b>Students can:</b></p> <p>a. Create equations that describe numbers or relationships. (CCSS: A-CED)</p> <ol style="list-style-type: none"> <li>i. Create equations and inequalities<sup>20</sup> in one variable and use them to solve problems. (CCSS: A-CED.1)</li> <li>ii. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: A-CED.2)</li> <li>iii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.<sup>21</sup> (CCSS: A-CED.3)</li> <li>iv. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.<sup>22</sup> (CCSS: A-CED.4)</li> </ol> <p>b. Understand solving equations as a process of reasoning and explain the reasoning. (CCSS: A-REI)</p> <ol style="list-style-type: none"> <li>i. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. (CCSS: A-REI.1)</li> <li>ii. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: A-REI.2)</li> </ol> <p>c. Solve equations and inequalities in one variable. (CCSS: A-REI)</p> <ol style="list-style-type: none"> <li>i. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS: A-REI.3)</li> <li>ii. Solve quadratic equations in one variable. (CCSS: A-REI.4) <ol style="list-style-type: none"> <li>1. Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form. (CCSS: A-REI.4a)</li> <li>2. Solve quadratic equations<sup>23</sup> by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. (CCSS: A-REI.4b)</li> <li>3. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>. (CCSS: A-REI.4b)</li> </ol> </li> </ol> <p>d. Solve systems of equations. (CCSS: A-REI)</p> <ol style="list-style-type: none"> <li>i. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: A-REI.5)</li> <li>ii. Solve systems of linear equations exactly and approximately,<sup>24</sup> focusing on pairs of linear equations in two variables. (CCSS: A-REI.6)</li> <li>iii. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.<sup>25</sup> (CCSS: A-REI.7)</li> </ol> <p>e. Represent and solve equations and inequalities graphically. (CCSS: A-REI)</p> <ol style="list-style-type: none"> <li>i. Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.<sup>26</sup> (CCSS: A-REI.10)</li> <li>ii. Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>;<sup>27</sup> find the solutions approximately.<sup>28*</sup> (CCSS: A-REI.11)</li> <li>iii. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: A-REI.12)</li> </ol>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>1. What are some similarities in solving all types of equations?</li> <li>2. Why do different types of equations require different types of solution processes?</li> <li>3. Can computers solve algebraic problems that people cannot solve? Why?</li> <li>4. How are order of operations and operational relationships important when solving multivariable equations?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>1. Linear programming allows representation of the constraints in a real-world situation identification of a feasible region and determination of the maximum or minimum value such as to optimize profit, or to minimize expense.</li> <li>2. Effective use of graphing technology helps to find solutions to equations or systems of equations.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematics involves visualization.</li> <li>2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning.</li> <li>3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>4. Mathematicians use appropriate tools strategically. (MP)</li> </ol>
<p><i>*Indicates a part of the standard connected to the mathematical practice of Modeling</i></p>	

## Standard: 2. Patterns, Functions, and Algebraic Structures High School

- <sup>1</sup> If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . (CCSS: F-IF.1)
- <sup>2</sup> For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ . (CCSS: F-IF.3)
- <sup>3</sup> Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS: F-IF.4)
- <sup>4</sup> For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS: F-IF.5)
- <sup>5</sup> presented symbolically or as a table. (CCSS: F-IF.6)
- <sup>6</sup> For example, identify percent rate of change in functions such as  $y = (1.02)t$ ,  $y = (0.97)t$ ,  $y = (1.01)12t$ ,  $y = (1.2)t/10$ ,. (CCSS: F-IF.8b)
- <sup>7</sup> For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS: F-IF.9)
- <sup>8</sup> For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (CCSS: F-BF.1b)
- <sup>9</sup> both positive and negative. (CCSS: F-BF.3)
- <sup>10</sup> Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS: F-BF.3)
- <sup>11</sup> Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ . (CCSS: F-BF.4a)
- <sup>12</sup> include reading these from a table. (CCSS: F-LE.2)
- <sup>13</sup> For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ . (CCSS: A-SSE.1b)
- <sup>14</sup> For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ . (CCSS: A-SSE.2)
- <sup>15</sup> For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (CCSS: A-SSE.3c)
- <sup>16</sup> For example, calculate mortgage payments. (CCSS: A-SSE.4)
- <sup>17</sup> For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ . (CCSS: A-APR.2)
- <sup>18</sup> For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples. (CCSS: A-APR.4)
- <sup>19</sup> write  $\frac{a(x)}{b(x)}$  in the form  $q(x) + \frac{r(x)}{b(x)}$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: A-APR.6)
- <sup>20</sup> Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (CCSS: A-CED.1)
- <sup>21</sup> For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (CCSS: A-CED.3)
- <sup>22</sup> For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ . (CCSS: A-CED.4)
- <sup>23</sup> e.g., for  $x^2 = 49$ . (CCSS: A-REI.4b)
- <sup>24</sup> e.g., with graphs. (CCSS: A-REI.6)
- <sup>25</sup> For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ . (CCSS: A-REI.7)
- <sup>26</sup> which could be a line. (CCSS: A-REI.10)
- <sup>27</sup> Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (CCSS: A-REI.11)
- <sup>28</sup> e.g., using technology to graph the functions, make tables of values, or find successive approximations. (CCSS: A-REI.11)



**Content Area: Mathematics**

**Standard: 2. Patterns, Functions, and Algebraic Structures**

**Prepared Graduates:**

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

**Grade Level Expectation: Eighth Grade**

**Concepts and skills students master:**

1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically

**Evidence Outcomes**

**Students can:**

- a. Describe the connections between proportional relationships, lines, and linear equations. (CCSS: 8.EE)
- b. Graph proportional relationships, interpreting the unit rate as the slope of the graph. (CCSS: 8.EE.5)
- c. Compare two different proportional relationships represented in different ways.<sup>1</sup> (CCSS: 8.EE.5)
- d. Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane. (CCSS: 8.EE.6)
- e. Derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ . (CCSS: 8.EE.6)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. How can different representations of linear patterns present different perspectives of situations?
2. How can a relationship be analyzed with tables, graphs, and equations?
3. Why is one variable dependent upon the other in relationships?

**Relevance and Application:**

1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates from competing companies or simple interest on savings or credit.
2. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related.
3. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function.

**Nature of Mathematics:**

1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model.
2. Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

#### Grade Level Expectation: Eighth Grade

##### Concepts and skills students master:

2. Properties of algebra and equality are used to solve linear equations and systems of equations

##### Evidence Outcomes

###### Students can:

- Solve linear equations in one variable. (CCSS: 8.EE.7)
  - Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.<sup>2</sup> (CCSS: 8.EE.7a)
  - Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.7b)
- Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.8)
  - Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.8a)
  - Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.<sup>3</sup> (CCSS: 8.EE.8b)
  - Solve real-world and mathematical problems leading to two linear equations in two variables.<sup>4</sup> (CCSS: 8.EE.8c)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- What makes a solution strategy both efficient and effective?
- How is it determined if multiple solutions to an equation are valid?
- How does the context of the problem affect the reasonableness of a solution?
- Why can two equations be added together to get another true equation?

###### Relevance and Application:

- The understanding and use of equations, inequalities, and systems of equations allows for situational analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs.
- Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs.

###### Nature of Mathematics:

- Mathematics involves visualization.
- Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians use appropriate tools strategically. (MP)

**Content Area: Mathematics**  
**Standard: 2. Patterns, Functions, and Algebraic Structures**

**Prepared Graduates:**  
 ➤ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Grade Level Expectation: Eighth Grade**

**Concepts and skills students master:**  
 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
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**Students can:**

- a. Define, evaluate, and compare functions. (CCSS: 8.F)
  - i. Define a function as a rule that assigns to each input exactly one output.<sup>5</sup> (CCSS: 8.F.1)
  - ii. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (CCSS: 8.F.1)
  - iii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<sup>6</sup> (CCSS: 8.F.2)
  - iv. Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line. (CCSS: 8.F.3)
  - v. Give examples of functions that are not linear.<sup>7</sup>
- b. Use functions to model relationships between quantities. (CCSS: 8.F)
  - i. Construct a function to model a linear relationship between two quantities. (CCSS: 8.F.4)
  - ii. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. (CCSS: 8.F.4)
  - iii. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.4)
  - iv. Describe qualitatively the functional relationship between two quantities by analyzing a graph.<sup>8</sup> (CCSS: 8.F.5)
  - v. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.5)
  - vi. Analyze how credit and debt impact personal financial goals (PFL)

**Inquiry Questions:**

1. How can change best be represented mathematically?
2. Why are patterns and relationships represented in multiple ways?
3. What properties of a function make it a linear function?

**Relevance and Application:**

1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation.
2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters.

**Nature of Mathematics:**

1. Mathematics involves multiple points of view.
2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches.
3. Mathematicians look for and make use of structure. (MP)
4. Mathematicians look for and express regularity in repeated reasoning. (MP)

## Standard: 2. Patterns, Functions, and Algebraic Structures Eighth Grade

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<sup>1</sup> For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS: 8.EE.5)

<sup>2</sup> Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers). (CCSS: 8.EE.6a)

<sup>3</sup> For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6. (CCSS: 8.EE.8b)

<sup>4</sup> For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (CCSS: 8.EE.8c)

<sup>5</sup> Function notation is not required in 8<sup>th</sup> grade. (CCSS: 8.F.1<sup>1</sup>)

<sup>6</sup> For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS: 8.F.2)

<sup>7</sup> For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points  $(1,1)$ ,  $(2,4)$  and  $(3,9)$ , which are not on a straight line. (CCSS: 8.F.3)

<sup>8</sup> e.g., where the function is increasing or decreasing, linear or nonlinear. (CCSS: 8.F.5)

**Content Area: Mathematics**

**Standard: 2. Patterns, Functions, and Algebraic Structures**

**Prepared Graduates:**  
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

**Grade Level Expectation: Seventh Grade**

**Concepts and skills students master:**  
1. Properties of arithmetic can be used to generate equivalent expressions

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <p>a. Use properties of operations to generate equivalent expressions. (CCSS: 7.EE)</p> <p>i. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.1)</p> <p>ii. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.<sup>1</sup> (CCSS: 7.EE.2)</p>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"><li>1. How do symbolic transformations affect an equation or expression?</li><li>2. How is it determined that two algebraic expressions are equivalent?</li></ol>
	<p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"><li>1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass.</li></ol>
	<p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"><li>1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions.</li><li>2. Mathematicians reason abstractly and quantitatively. (MP)</li><li>3. Mathematicians look for and express regularity in repeated reasoning. (MP)</li></ol>

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

#### Grade Level Expectation: Seventh Grade

#### Concepts and skills students master:

2. Equations and expressions model quantitative relationships and phenomena

#### Evidence Outcomes

##### Students can:

- a. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form,<sup>2</sup> using tools strategically. (CCSS: 7.EE.3)
- b. Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.<sup>3</sup> (CCSS: 7.EE.3)
- c. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.4)
  - i. Fluently solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. (CCSS: 7.EE.4a)
  - ii. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.<sup>4</sup> (CCSS: 7.EE.4a)
  - iii. Solve word problems<sup>5</sup> leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. (CCSS: 7.EE.4b)
  - iv. Graph the solution set of the inequality and interpret it in the context of the problem. (CCSS: 7.EE.4b)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

1. Do algebraic properties work with numbers or just symbols? Why?
2. Why are there different ways to solve equations?
3. How are properties applied in other fields of study?
4. Why might estimation be better than an exact answer?
5. When might an estimate be the only possible answer?

##### Relevance and Application:

1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed.
2. Comprehension of the structure of equations allows one to use spreadsheets effectively to solve problems that matter such as showing how long it takes to pay off debt, or representing data collected from science experiments.
3. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard.

##### Nature of Mathematics:

1. Mathematicians model with mathematics. (MP)

**Standard: 2. Patterns, Functions, and Algebraic Structures**  
**Seventh Grade**

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<sup>1</sup> For example,  $a + 0.05a = 1.05a$  means that “increase by 5%” is the same as “multiply by 1.05.” (CCSS: 7.EE.2)

<sup>2</sup> whole numbers, fractions, and decimals. (CCSS: 7.EE.3)

<sup>3</sup> For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional  $\frac{1}{10}$  of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. (CCSS: 7.EE.3)

<sup>4</sup> For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? (CCSS: 7.EE.4a)

<sup>5</sup> For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions. (CCSS: 7.EE.4b)

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: Sixth Grade

##### Concepts and skills students master:

1. Algebraic expressions can be used to generalize properties of arithmetic

##### Evidence Outcomes

###### Students can:

- a. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.1)
- b. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.2)
  - i. Write expressions that record operations with numbers and with letters standing for numbers.<sup>1</sup> (CCSS: 6.EE.2a)
  - ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity.<sup>2</sup> (CCSS: 6.EE.2b)
  - iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems.<sup>3</sup> (CCSS: 6.EE.2c)
  - iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (CCSS: 6.EE.2c)
- c. Apply the properties of operations to generate equivalent expressions.<sup>4</sup> (CCSS: 6.EE.3)
- d. Identify when two expressions are equivalent.<sup>5</sup> (CCSS: 6.EE.4)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. If we didn't have variables, what would we use?
2. What purposes do variable expressions serve?
3. What are some advantages to being able to describe a pattern using variables?
4. Why does the order of operations exist?
5. What other tasks/processes require the use of a strict order of steps?

###### Relevance and Application:

1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts.
2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies.

###### Nature of Mathematics:

1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships.
2. Mathematics can be expressed in a variety of formats.
3. Mathematicians reason abstractly and quantitatively. (MP)
4. Mathematicians look for and make use of structure. (MP)
5. Mathematicians look for and express regularity in repeated reasoning. (MP)



## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: Sixth Grade

##### Concepts and skills students master:

2. Variables are used to represent unknown quantities within equations and inequalities

##### Evidence Outcomes

###### Students can:

- Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (CCSS: 6.EE.5)
- Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.5)
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. (CCSS: 6.EE.6)
  - Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.6)
- Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$  and  $x$  are all nonnegative rational numbers. (CCSS: 6.EE.7)
- Write an inequality of the form  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world or mathematical problem. (CCSS: 6.EE.8)
- Show that inequalities of the form  $x > c$  or  $x < c$  have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.8)
- Represent and analyze quantitative relationships between dependent and independent variables. (CCSS: 6.EE.9)
  - Use variables to represent two quantities in a real-world problem that change in relationship to one another. (CCSS: 6.EE.9)
  - Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (CCSS: 6.EE.9)
  - Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.<sup>6</sup> (CCSS: 6.EE.9)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- Do all equations have exactly one unique solution? Why?
- How can you determine if a variable is independent or dependent?

###### Relevance and Application:

- Variables allow communication of big ideas with very few symbols. For example,  $d = r * t$  is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and  $C = \pi d$  expresses the relationship between circumference and diameter of a circle.
- Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change.

###### Nature of Mathematics:

- Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables.
- Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable.
- Mathematicians model with mathematics. (MP)

**Standard: 2. Patterns, Functions, and Algebraic Structures**  
**Sixth Grade**

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<sup>1</sup> For example, express the calculation “Subtract  $y$  from 5” as  $5 - y$ . (CCSS: 6.EE.2a)

<sup>2</sup> For example, describe the expression  $2(8 + 7)$  as a product of two factors; view  $(8 + 7)$  as both a single entity and a sum of two terms. (CCSS: 6.EE.2b)

<sup>3</sup> For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ . (CCSS: 6.EE.2c)

<sup>4</sup> For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ . (CCSS: 6.EE.3)

<sup>5</sup> i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for. Reason about and solve one-variable equations and inequalities. (CCSS: 6.EE.4)

<sup>6</sup> For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation  $d = 65t$  to represent the relationship between distance and time. (CCSS: 6.EE.9)

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

#### Grade Level Expectation: Fifth Grade

##### Concepts and skills students master:

1. Number patterns are based on operations and relationships

##### Evidence Outcomes

###### Students can:

- a. Generate two numerical patterns using given rules. (CCSS: 5.OA.3)
- b. Identify apparent relationships between corresponding terms. (CCSS: 5.OA.3)
- c. Form ordered pairs consisting of corresponding terms from the two patterns, and graphs the ordered pairs on a coordinate plane.<sup>1</sup> (CCSS: 5.OA.3)
- d. Explain informally relationships between corresponding terms in the patterns. (CCSS: 5.OA.3)
- e. Use patterns to solve problems including those involving saving and checking accounts<sup>2</sup> (PFL)
- f. Explain, extend, and use patterns and relationships in solving problems, including those involving saving and checking accounts such as understanding that spending more means saving less (PFL)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. How do you know when there is a pattern?
2. How are patterns useful?

###### Relevance and Application:

1. The use of a pattern of elapsed time helps to set up a schedule. For example, classes are each 50 minutes with 5 minutes between each class.
2. The ability to use patterns allows problem-solving. For example, a rancher needs to know how many shoes to buy for his horses, or a grocer needs to know how many cans will fit on a set of shelves.

###### Nature of Mathematics:

1. Mathematicians use creativity, invention, and ingenuity to understand and create patterns.
2. The search for patterns can produce rewarding shortcuts and mathematical insights.
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
4. Mathematicians model with mathematics. (MP)
5. Mathematicians look for and express regularity in repeated reasoning. (MP)

**Standard: 2. Patterns, Functions, and Algebraic Structures**  
**Fifth Grade**

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<sup>1</sup> For example, given the rule “add 3” and the starting number 0, and given the rule “add 6” and the starting number 0, generate terms and the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. (CCSS: 5.OA.3)

<sup>2</sup> such as the pattern created when saving \$10 a month

## Content Area: Mathematics

### Standard: 2. Patterns, Functions, and Algebraic Structures

#### Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

##### Evidence Outcomes

###### Students can:

- a. Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself.<sup>1</sup> (CCSS: 4.OA.5)
  - i. Use number relationships to find the missing number in a sequence
  - ii. Use a symbol to represent and find an unknown quantity in a problem situation
  - iii. Complete input/output tables
  - iv. Find the unknown in simple equations
- b. Apply concepts of squares, primes, composites, factors, and multiples to solve problems
  - i. Find all factor pairs for a whole number in the range 1–100. (CCSS: 4.OA.4)
  - ii. Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)
  - iii. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. (CCSS: 4.OA.4)
  - iv. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.4)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. What characteristics can be used to classify numbers into different groups?
2. How can we predict the next element in a pattern?
3. Why do we use symbols to represent missing numbers?
4. Why is finding an unknown quantity important?

###### Relevance and Application:

1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth.
2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school.
3. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators.

###### Nature of Mathematics:

1. Mathematics involves pattern seeking.
2. Mathematicians use patterns to simplify calculations.
3. Mathematicians model with mathematics. (MP)

**Standard: 2. Patterns, Functions, and Algebraic Structures**  
**Fourth Grade**

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<sup>1</sup> For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.5)

**Content Area: Mathematics**

**Standard: 2. Patterns, Functions, and Algebraic Structures**

**Prepared Graduates:**

**Grade Level Expectation: PRESCHOOL THROUGH THIRD GRADE**

**Concepts and skills students master:**

**Evidence Outcomes**

**Students can:**

**Expectations for this standard are integrated into the other standards at preschool through third grade.**

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

**Relevance and Application:**

**Nature of Mathematics:**

# 3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

## Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

**Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:**

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions



**Content Area: Mathematics**

**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: High School**

**Concepts and skills students master:**

1. Visual displays and summary statistics condense the information in data sets into usable knowledge

**Evidence Outcomes**

**Students can:**

- a. Summarize, represent, and interpret data on a single count or measurement variable. (CCSS: S-ID)
  - i. Represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS: S-ID.1)
  - ii. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: S-ID.2)
  - iii. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: S-ID.3)
  - iv. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. (CCSS: S-ID.4)
  - v. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: S-ID.4)
- b. Summarize, represent, and interpret data on two categorical and quantitative variables. (CCSS: S-ID)
  - i. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data<sup>1</sup> (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: S-ID.5)
  - ii. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: S-ID.6)
    1. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (CCSS: S-ID.6a)
    2. Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: S-ID.6b)
    3. Fit a linear function for a scatter plot that suggests a linear association. (CCSS: S-ID.6c)
- c. Interpret linear models. (CCSS: S-ID)
  - i. Interpret the slope<sup>2</sup> and the intercept<sup>3</sup> of a linear model in the context of the data. (CCSS: S-ID.7)
  - ii. Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: S-ID.8)
  - iii. Distinguish between correlation and causation. (CCSS: S-ID.9)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What makes data meaningful or actionable?
2. Why should attention be paid to an unexpected outcome?
3. How can summary statistics or data displays be accurate but misleading?

**Relevance and Application:**

1. Facility with data organization, summary, and display allows the sharing of data efficiently and collaboratively to answer important questions such as is the climate changing, how do people think about ballot initiatives in the next election, or is there a connection between cancers in a community?

**Nature of Mathematics:**

1. Mathematicians create visual and numerical representations of data to reveal relationships and meaning hidden in the raw data.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians model with mathematics. (MP)
4. Mathematicians use appropriate tools strategically. (MP)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**  
 ➤ Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking

**Grade Level Expectation: High School**

**Concepts and skills students master:**  
 2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
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**Students can:**

- a. Understand and evaluate random processes underlying statistical experiments. (CCSS: S-IC)
  - i. Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: S-IC.1)
  - ii. Decide if a specified model is consistent with results from a given data-generating process.<sup>4</sup> (CCSS: S-IC.2)
- b. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (CCSS: S-IC)
  - i. Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: S-IC.3)
  - ii. Use data from a sample survey to estimate a population mean or proportion. (CCSS: S-IC.4)
  - iii. Develop a margin of error through the use of simulation models for random sampling. (CCSS: S-IC.4)
  - iv. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: S-IC.5)
  - v. Define and explain the meaning of significance, both statistical (using  $p$ -values) and practical (using effect size).
  - vi. Evaluate reports based on data. (CCSS: S-IC.6)

**Inquiry Questions:**

1. How can the results of a statistical investigation be used to support an argument?
2. What happens to sample-to-sample variability when you increase the sample size?
3. When should sampling be used? When is sampling better than using a census?
4. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference?
5. Why is the margin of error in a study important?
6. How is it known that the results of a study are not simply due to chance?

**Relevance and Application:**

1. Inference and prediction skills enable informed decision-making based on data such as whether to stop using a product based on safety concerns, or whether a political poll is pointing to a trend.

**Nature of Mathematics:**

1. Mathematics involves making conjectures, gathering data, recording results, and making multiple tests.
2. Mathematicians are skeptical of apparent trends. They use their understanding of randomness to distinguish meaningful trends from random occurrences.
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
4. Mathematicians model with mathematics. (MP)
5. Mathematicians attend to precision. (MP)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

**Grade Level Expectation: High School**

**Concepts and skills students master:**

3. Probability models outcomes for situations in which there is inherent randomness

**Evidence Outcomes**

**Students can:**

- a. Understand independence and conditional probability and use them to interpret data. (CCSS: S-CP)
  - i. Describe events as subsets of a sample space<sup>5</sup> using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.<sup>6</sup> (CCSS: S-CP.1)
  - ii. Explain that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: S-CP.2)
  - iii. Using the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , interpret the independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ . (CCSS: S-CP.3)
  - iv. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.<sup>7</sup> (CCSS: S-CP.4)
  - v. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.<sup>8</sup> (CCSS: S-CP.5)
- b. Use the rules of probability to compute probabilities of compound events in a uniform probability model. (CCSS: S-CP)
  - i. Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model. (CCSS: S-CP.6)
  - ii. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. (CCSS: S-CP.7)
- c. Analyze\* the cost of insurance as a method to offset the risk of a situation (PFL)

\*Indicates a part of the standard connected to the mathematical practice of Modeling.

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. Can probability be used to model all types of uncertain situations? For example, can the probability that the 50<sup>th</sup> president of the United States will be female be determined?
2. How and why are simulations used to determine probability when the theoretical probability is unknown?
3. How does probability relate to obtaining insurance? (PFL)

**Relevance and Application:**

1. Comprehension of probability allows informed decision-making, such as whether the cost of insurance is less than the expected cost of illness, when the deductible on car insurance is optimal, whether gambling pays in the long run, or whether an extended warranty justifies the cost. (PFL)
2. Probability is used in a wide variety of disciplines including physics, biology, engineering, finance, and law. For example, employment discrimination cases often present probability calculations to support a claim.

**Nature of Mathematics:**

1. Some work in mathematics is much like a game. Mathematicians choose an interesting set of rules and then play according to those rules to see what can happen.
2. Mathematicians explore randomness and chance through probability.
3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
4. Mathematicians model with mathematics. (MP)

### **Standard: 3. Data Analysis, Statistics, and Probability High School**

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<sup>1</sup> including joint, marginal, and conditional relative frequencies.

<sup>2</sup> rate of change. (CCSS: S-ID.7)

<sup>3</sup> constant term. (CCSS: S-ID.7)

<sup>4</sup> e.g., using simulation. (CCSS: S-IC.2)

For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (CCSS: S-IC.2)

<sup>5</sup> the set of outcomes. (CCSS: S-CP.1)

<sup>6</sup> "or," "and," "not". (CCSS: S-CP.1)

<sup>7</sup> For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English.

Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (CCSS: S-CP.4)

<sup>8</sup> For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (CCSS: S-CP.5)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Eighth Grade**

**Concepts and skills students master:**

1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ol style="list-style-type: none"> <li>a. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. (CCSS: 8.SP.1)</li> <li>b. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.1)</li> <li>c. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.<sup>1</sup> (CCSS: 8.SP.2)</li> <li>d. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.<sup>2</sup> (CCSS: 8.SP.3)</li> <li>e. Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. (CCSS: 8.SP.4)               <ol style="list-style-type: none"> <li>i. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. (CCSS: 8.SP.4)</li> <li>ii. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.<sup>3</sup> (CCSS: 8.SP.4)</li> </ol> </li> </ol>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>1. How is it known that two variables are related to each other?</li> <li>2. How is it known that an apparent trend is just a coincidence?</li> <li>3. How can correct data lead to incorrect conclusions?</li> <li>4. How do you know when a credible prediction can be made?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>1. The ability to analyze and interpret data helps to distinguish between false relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation.</li> <li>2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predicting whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematicians discover new relationship embedded in information.</li> <li>2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>3. Mathematicians model with mathematics. (MP)</li> </ol>

**Standard: 3. Data Analysis, Statistics, and Probability**  
**Eighth Grade**

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<sup>1</sup> Know that straight lines are widely used to model relationships between two quantitative variables. (CCSS: 8.SP.2)

<sup>2</sup> For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (CCSS: 8.SP.3)

<sup>3</sup> For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS: 8.SP.4)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Grade Level Expectation: Seventh Grade**

**Concepts and skills students master:**

1. Statistics can be used to gain information about populations by examining samples

**Evidence Outcomes**

**Students can:**

- a. Use random sampling to draw inferences about a population. (CCSS: 7.SP)
  - i. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population. (CCSS: 7.SP.1)
  - ii. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.1)
  - iii. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. (CCSS: 7.SP.2)
  - iv. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.<sup>1</sup> (CCSS: 7.SP.2)
- b. Draw informal comparative inferences about two populations. (CCSS: 7.SP)
  - i. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.<sup>2</sup> (CCSS: 7.SP.3)
  - ii. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.<sup>3</sup> (CCSS: 7.SP.4)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. How might the sample for a survey affect the results of the survey?
2. How do you distinguish between random and bias samples?
3. How can you declare a winner in an election before counting all the ballots?

**Relevance and Application:**

1. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness or show strong opinions.
2. Mathematical inferences allow us to make reliable predictions without accounting for every piece of data.

**Nature of Mathematics:**

1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions.
2. Mathematicians use appropriate tools strategically. (MP)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

**Grade Level Expectation: Seventh Grade**

**Concepts and skills students master:**  
 2. Mathematical models are used to determine probability

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ul style="list-style-type: none"> <li>a. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.<sup>4</sup> (CCSS: 7.SP.5)</li> <li>b. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.<sup>5</sup> (CCSS: 7.SP.6)</li> <li>c. Develop a probability model and use it to find probabilities of events. (CCSS: 7.SP.7)               <ul style="list-style-type: none"> <li>i. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.7)</li> <li>ii. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.<sup>6</sup> (CCSS: 7.SP.7a)</li> <li>iii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.<sup>7</sup> (CCSS: 7.SP.7b)</li> </ul> </li> <li>d. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.8)               <ul style="list-style-type: none"> <li>i. Explain that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.8a)</li> <li>ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. (CCSS: 7.SP.8b)</li> <li>iii. For an event<sup>8</sup> described in everyday language identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.8b)</li> <li>iv. Design and use a simulation to generate frequencies for compound events.<sup>9</sup> (CCSS: 7.SP.8c)</li> </ul> </li> </ul>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>1. Why is it important to consider all of the possible outcomes of an event?</li> <li>2. Is it possible to predict the future? How?</li> <li>3. What are situations in which probability cannot be used?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers.</li> <li>2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes.</li> <li>2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)</li> <li>3. Mathematicians model with mathematics. (MP)</li> </ol>



### **Standard: 3. Data Analysis, Statistics, and Probability Seventh Grade**

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<sup>1</sup> For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS: 7.SP.2)

<sup>2</sup> For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS: 7.SP.3)

<sup>3</sup> For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS: 7.SP.4)

<sup>4</sup> Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.5)

<sup>5</sup> For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS: 7.SP.6)

<sup>6</sup> For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. (CCSS: 7.SP.7a)

<sup>7</sup> For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS: 7.SP.7b)

<sup>8</sup> e.g., "rolling double sixes" (CCSS: 7.SP.8b)

<sup>9</sup> For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? (CCSS: 7.SP.8c)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Sixth Grade**

**Concepts and skills students master:**

1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ol style="list-style-type: none"> <li>a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.<sup>1</sup> (CCSS: 6.SP.1)</li> <li>b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS: 6.SP.2)</li> <li>c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.3)</li> <li>d. Summarize and describe distributions. (CCSS: 6.SP)               <ol style="list-style-type: none"> <li>i. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.4)</li> <li>ii. Summarize numerical data sets in relation to their context. (CCSS: 6.SP.5)                   <ol style="list-style-type: none"> <li>1. Report the number of observations. (CCSS: 6.SP.5a)</li> <li>2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.5b)</li> <li>3. Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.5c)</li> <li>4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.5d)</li> </ol> </li> </ol> </li> </ol>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>1. Why are there so many ways to describe data?</li> <li>2. When is one data display better than another?</li> <li>3. When is one statistical measure better than another?</li> <li>4. What makes a good statistical question?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system.</li> <li>2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>1. Mathematicians leverage strategic displays to reveal data.</li> <li>2. Mathematicians model with mathematics. (MP)</li> <li>3. Mathematicians use appropriate tools strategically. (MP)</li> <li>4. Mathematicians attend to precision. (MP)</li> </ol>

**Standard: 3. Data Analysis, Statistics, and Probability**  
**Sixth Grade**

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<sup>1</sup> For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages. (CCSS: 6.SP.1)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**  
 ➤ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Fifth Grade**

**Concepts and skills students master:**  
 1. Visual displays are used to interpret data

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <p>a. Represent and interpret data. (CCSS: 5.MD)</p> <p>    i. Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). (CCSS: 5.MD.2)</p> <p>    ii. Use operations on fractions for this grade to solve problems involving information presented in line plots.<sup>1</sup> (CCSS: 5.MD.2)</p>	<p><b>Inquiry Questions:</b></p> <p>1. How can you make sense of the data you collect?</p>
	<p><b>Relevance and Application:</b></p> <p>1. The collection and analysis of data provides understanding of how things work. For example, measuring the temperature every day for a year helps to better understand weather.</p>
	<p><b>Nature of Mathematics:</b></p> <p>1. Mathematics helps people collect and use information to make good decisions.</p> <p>2. Mathematicians model with mathematics. (MP)</p> <p>3. Mathematicians use appropriate tools strategically. (MP)</p> <p>4. Mathematicians attend to precision. (MP)</p>

**Standard: 3. Data Analysis, Statistics, and Probability**  
**Fifth Grade**

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<sup>1</sup> For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS: 5.MD.2)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**  
 ➤ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Fourth Grade**

**Concepts and skills students master:**  
 1. Visual displays are used to represent data

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ol style="list-style-type: none"> <li>Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). (CCSS: 4.MD.4)</li> <li>Solve problems involving addition and subtraction of fractions by using information presented in line plots.<sup>1</sup> (CCSS: 4.MD.4)</li> </ol>	<p><b>Inquiry Questions:</b></p> <ol style="list-style-type: none"> <li>What can you learn by collecting data?</li> <li>What can the shape of data in a display tell you?</li> </ol> <p><b>Relevance and Application:</b></p> <ol style="list-style-type: none"> <li>The collection and analysis of data provides understanding of how things work. For example, measuring the weather every day for a year helps to better understand weather.</li> </ol> <p><b>Nature of Mathematics:</b></p> <ol style="list-style-type: none"> <li>Mathematics helps people use data to learn about the world.</li> <li>Mathematicians model with mathematics. (MP)</li> <li>Mathematicians use appropriate tools strategically. (MP)</li> <li>Mathematicians attend to precision. (MP)</li> </ol>

**Standard: 3. Data Analysis, Statistics, and Probability**  
**Fourth Grade**

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<sup>1</sup> For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.  
(CCSS: 4.MD.4)

**Content Area: Mathematics**

**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

1. Visual displays are used to describe data

**Evidence Outcomes**

**Students can:**

- a. Represent and interpret data. (CCSS: 3.MD)
  - i. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. (CCSS: 3.MD.3)
  - ii. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.<sup>1</sup> (CCSS: 3.MD.3)
  - iii. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. (CCSS: 3.MD.4)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What can data tell you about your class or school?
2. How do data displays help us understand information?

**Relevance and Application:**

1. The collection and use of data provides better understanding of people and the world such as knowing what games classmates like to play, how many siblings friends have, or personal progress made in sports.

**Nature of Mathematics:**

1. Mathematical data can be represented in both static and animated displays.
2. Mathematicians model with mathematics. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)



**Standard: 3. Data Analysis, Statistics, and Probability**  
**Third Grade**

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<sup>1</sup> For example, draw a bar graph in which each square in the bar graph might represent 5 pets. (CCSS: 3.MD.3)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: Second Grade**

**Concepts and skills students master:**

1. Visual displays of data can be constructed in a variety of formats to solve problems

**Evidence Outcomes**

**Students can:**

- a. Represent and interpret data. (CCSS: 2.MD)
  - i. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.9)
  - ii. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. (CCSS: 2.MD.10)
  - iii. Solve simple put together, take-apart, and compare problems using information presented in picture and bar graphs. (CCSS: 2.MD.10)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What are the ways data can be displayed?
2. What can data tell you about the people you survey?
3. What makes a good survey question?

**Relevance and Application:**

1. People use data to describe the world and answer questions such as how many classmates are buying lunch today, how much it rained yesterday, or in which month are the most birthdays.

**Nature of Mathematics:**

1. Mathematics can be displayed as symbols.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians model with mathematics. (MP)
4. Mathematicians attend to precision. (MP)

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**  
 ➤ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

**Grade Level Expectation: First Grade**

**Concepts and skills students master:**  
 1. Visual displays of information can used to answer questions

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ul style="list-style-type: none"> <li>a. Represent and interpret data. (CCSS: 1.MD)               <ul style="list-style-type: none"> <li>i. Organize, represent, and interpret data with up to three categories. (CCSS: 1.MD.4)</li> <li>ii. Ask and answer questions about the total number of data points how many in each category, and how many more or less are in one category than in another. (CCSS: 1.MD.4)</li> </ul> </li> </ul>	<p><b>Inquiry Questions:</b></p> <ul style="list-style-type: none"> <li>1. What kinds of questions generate data?</li> <li>2. What questions can be answered by a data representation?</li> </ul> <p><b>Relevance and Application:</b></p> <ul style="list-style-type: none"> <li>1. People use graphs and charts to communicate information and learn about a class or community such as the kinds of cars people drive, or favorite ice cream flavors of a class.</li> </ul> <p><b>Nature of Mathematics:</b></p> <ul style="list-style-type: none"> <li>1. Mathematicians organize and explain random information</li> <li>2. Mathematicians model with mathematics. (MP)</li> </ul>

**Content Area: Mathematics**  
**Standard: 3. Data Analysis, Statistics, and Probability**

**Prepared Graduates:**

**Grade Level Expectation: PRESCHOOL AND KINDERGARTEN**

**Concepts and skills students master:**

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
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**Students can:**

**Expectations for this standard are integrated into the other standards at preschool through kindergarten.**

**Inquiry Questions:**

**Relevance and Application:**

**Nature of Mathematics:**

# 4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

## Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

### Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Content Area: Mathematics****Standard: 4. Shape, Dimension, and Geometric Relationships****Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: High School****Concepts and skills students master:**

1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

**Evidence Outcomes****Students can:**

- a. Experiment with transformations in the plane. (CCSS: G-CO)
  - i. State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: G-CO.1)
  - ii. Represent transformations in the plane using<sup>1</sup> appropriate tools. (CCSS: G-CO.2)
  - iii. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. (CCSS: G-CO.2)
  - iv. Compare transformations that preserve distance and angle to those that do not.<sup>2</sup> (CCSS: G-CO.2)
  - v. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS: G-CO.3)
  - vi. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: G-CO.4)
  - vii. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools.<sup>3</sup> (CCSS: G-CO.5)
  - viii. Specify a sequence of transformations that will carry a given figure onto another. (CCSS: G-CO.5)
- b. Understand congruence in terms of rigid motions. (CCSS: G-CO)
  - i. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. (CCSS: G-CO.6)
  - ii. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: G-CO.6)
  - iii. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: G-CO.7)
  - iv. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: G-CO.8)
- c. Prove geometric theorems. (CCSS: G-CO)
  - i. Prove theorems about lines and angles.<sup>4</sup> (CCSS: G-CO.9)
  - ii. Prove theorems about triangles.<sup>5</sup> (CCSS: G-CO.10)
  - iii. Prove theorems about parallelograms.<sup>6</sup> (CCSS: G-CO.11)
- d. Make geometric constructions. (CCSS: G-CO)
  - i. Make formal geometric constructions<sup>7</sup> with a variety of tools and methods.<sup>8</sup> (CCSS: G-CO.12)
  - ii. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: G-CO.13)

**21<sup>st</sup> Century Skills and Readiness Competencies****Inquiry Questions:**

1. What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane?
2. How would the idea of congruency be used outside of mathematics?
3. What does it mean for two things to be the same? Are there different degrees of "sameness?"
4. What makes a good definition of a shape?

**Relevance and Application:**

1. Comprehension of transformations aids with innovation and creation in the areas of computer graphics and animation.

**Nature of Mathematics:**

1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians attend to precision. (MP)
4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics****Standard: 4. Shape, Dimension, and Geometric Relationships****Prepared Graduates:**

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

**Grade Level Expectation: High School****Concepts and skills students master:**

2. Concepts of similarity are foundational to geometry and its applications

**Evidence Outcomes****Students can:**

- a. Understand similarity in terms of similarity transformations. (CCSS: G-SRT)
  - i. Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: G-SRT.1)
    1. Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: G-SRT.1a)
    2. Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: G-SRT.1b)
  - ii. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar. (CCSS: G-SRT.2)
  - iii. Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: G-SRT.2)
  - iv. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: G-SRT.3)
- b. Prove theorems involving similarity. (CCSS: G-SRT)
  - i. Prove theorems about triangles.<sup>9</sup> (CCSS: G-SRT.4)
  - ii. Prove that all circles are similar. (CCSS: G-C.1)
  - iii. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: G-SRT.5)
- c. Define trigonometric ratios and solve problems involving right triangles. (CCSS: G-SRT)
  - i. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: G-SRT.6)
  - ii. Explain and use the relationship between the sine and cosine of complementary angles. (CCSS: G-SRT.7)
  - iii. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\* (CCSS: G-SRT.8)
- d. Prove and apply trigonometric identities. (CCSS: F-TF)
  - i. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$ . (CCSS: F-TF.8)
  - ii. Use the Pythagorean identity to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle. (CCSS: F-TF.8)
- e. Understand and apply theorems about circles. (CCSS: G-C)
  - i. Identify and describe relationships among inscribed angles, radii, and chords.<sup>10</sup> (CCSS: G-C.2)
  - ii. Construct the inscribed and circumscribed circles of a triangle. (CCSS: G-C.3)
  - iii. Prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: G-C.3)
- f. Find arc lengths and areas of sectors of circles. (CCSS: G-C)
  - i. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality. (CCSS: G-C.5)
  - ii. Derive the formula for the area of a sector. (CCSS: G-C.5)

\*Indicates a part of the standard connected to the mathematical practice of Modeling

**21<sup>st</sup> Century Skills and Readiness Competencies****Inquiry Questions:**

1. How can you determine the measure of something that you cannot measure physically?
2. How is a corner square made?
3. How are mathematical triangles different from triangles in the physical world? How are they the same?
4. Do perfect circles naturally occur in the physical world?

**Relevance and Application:**

1. Analyzing geometric models helps one understand complex physical systems. For example, modeling Earth as a sphere allows us to calculate measures such as diameter, circumference, and surface area. We can also model the solar system, galaxies, molecules, atoms, and subatomic particles.

**Nature of Mathematics:**

1. Geometry involves the generalization of ideas. Geometers seek to understand and describe what is true about all cases related to geometric phenomena.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians attend to precision. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: High School

##### Concepts and skills students master:

3. Objects in the plane can be described and analyzed algebraically

##### Evidence Outcomes

###### Students can:

- Express Geometric Properties with Equations. (CCSS: G-GPE)
  - Translate between the geometric description and the equation for a conic section. (CCSS: G-GPE)
    - Derive the equation of a circle of given center and radius using the Pythagorean Theorem. (CCSS: G-GPE.1)
    - Complete the square to find the center and radius of a circle given by an equation. (CCSS: G-GPE.1)
    - Derive the equation of a parabola given a focus and directrix. (CCSS: G-GPE.2)
  - Use coordinates to prove simple geometric theorems algebraically. (CCSS: G-GPE)
    - Use coordinates to prove simple geometric theorems<sup>11</sup> algebraically. (CCSS: G-GPE.4)
    - Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.<sup>12</sup> (CCSS: G-GPE.5)
    - Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: G-GPE.6)
    - Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles.\* (CCSS: G-GPE.7)

*\*Indicates a part of the standard connected to the mathematical practice of Modeling*

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- What does it mean for two lines to be parallel?
- What happens to the coordinates of the vertices of shapes when different transformations are applied in the plane?

###### Relevance and Application:

- Knowledge of right triangle trigonometry allows modeling and application of angle and distance relationships such as surveying land boundaries, shadow problems, angles in a truss, and the design of structures.

###### Nature of Mathematics:

- Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)



## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: High School

#### Concepts and skills students master:

4. Attributes of two- and three-dimensional objects are measurable and can be quantified

#### Evidence Outcomes

##### Students can:

- Explain volume formulas and use them to solve problems. (CCSS: G-GMD)
  - Give an informal argument<sup>13</sup> for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. (CCSS: G-GMD.1)
  - Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.\* (CCSS: G-GMD.3)
- Visualize relationships between two-dimensional and three-dimensional objects. (CCSS: G-GMD)
  - Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: G-GMD.4)

*\*Indicates a part of the standard connected to the mathematical practice of Modeling*

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

- How might surface area and volume be used to explain biological differences in animals?
- How is the area of an irregular shape measured?
- How can surface area be minimized while maximizing volume?

##### Relevance and Application:

- Understanding areas and volume enables design and building. For example, a container that maximizes volume and minimizes surface area will reduce costs and increase efficiency. Understanding area helps to decorate a room, or create a blueprint for a new building.

##### Nature of Mathematics:

- Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

#### Grade Level Expectation: High School

##### Concepts and skills students master:

5. Objects in the real world can be modeled using geometric concepts

##### Evidence Outcomes

###### Students can:

- Apply geometric concepts in modeling situations. (CCSS: G-MG)
  - Use geometric shapes, their measures, and their properties to describe objects.<sup>14\*</sup> (CCSS: G-MG.1)
  - Apply concepts of density based on area and volume in modeling situations.<sup>15\*</sup> (CCSS: G-MG.2)
  - Apply geometric methods to solve design problems.<sup>16\*</sup> (CCSS: G-MG.3)

*\*Indicates a part of the standard connected to the mathematical practice of Modeling*

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- How are mathematical objects different from the physical objects they model?
- What makes a good geometric model of a physical object or situation?
- How are mathematical triangles different from built triangles in the physical world? How are they the same?

###### Relevance and Application:

- Geometry is used to create simplified models of complex physical systems. Analyzing the model helps to understand the system and is used for such applications as creating a floor plan for a house, or creating a schematic diagram for an electrical system.

###### Nature of Mathematics:

- Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden.
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians reason abstractly and quantitatively. (MP)
- Mathematicians look for and make use of structure. (MP)

## Standard: 4. Shape, Dimension, and Geometric Relationships High School

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<sup>1</sup> e.g., transparencies and geometry software. (CCSS: G-CO.2)

<sup>2</sup> e.g., translation versus horizontal stretch. (CCSS: G-CO.2)

<sup>3</sup> e.g., graph paper, tracing paper, or geometry software. (CCSS: G-CO.5)

<sup>4</sup> Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (CCSS: G-CO.9)

<sup>5</sup> Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (CCSS: G-CO.10)

<sup>6</sup> Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (CCSS: G-CO.11)

<sup>7</sup> Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (CCSS: G-CO.12)

<sup>8</sup> compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc. (CCSS: G-CO.12)

<sup>9</sup> Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (CCSS: G-SRT.4)

<sup>10</sup> Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (CCSS: G-C.2)

<sup>11</sup> For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ . (CCSS: G-GPE.4)

<sup>12</sup> e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. (CCSS: G-GPE.5)

<sup>13</sup> Use dissection arguments, Cavalieri's principle, and informal limit arguments. (CCSS: G-GMD.1)

<sup>14</sup> e.g., modeling a tree trunk or a human torso as a cylinder. (CCSS: G-MG.1)

<sup>15</sup> e.g., persons per square mile, BTUs per cubic foot. (CCSS: G-MG.2)

<sup>16</sup> e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. (CCSS: G-MG.3)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: Eighth Grade**

**Concepts and skills students master:**

1. Transformations of objects can be used to define the concepts of congruence and similarity

**Evidence Outcomes**

**Students can:**

- a. Verify experimentally the properties of rotations, reflections, and translations.<sup>1</sup> (CCSS: 8.G.1)
- b. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.3)
- c. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. (CCSS: 8.G.2)
- d. Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. (CCSS: 8.G.2)
- e. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. (CCSS: 8.G.4)
- f. Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. (CCSS: 8.G.4)
- g. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.<sup>2</sup> (CCSS: 8.G.5)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes?
2. How can you physically verify that two lines are really parallel?

**Relevance and Application:**

1. Dilations are used to enlarge or shrink pictures.
2. Rigid motions can be used to make new patterns for clothing or architectural design.

**Nature of Mathematics:**

1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

#### Grade Level Expectation: Eighth Grade

##### Concepts and skills students master:

2. Direct and indirect measurement can be used to describe and make comparisons

##### Evidence Outcomes

###### Students can:

- Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.6)
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.7)
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.8)
- State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.9)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- Why does the Pythagorean Theorem only apply to right triangles?
- How can the Pythagorean Theorem be used for indirect measurement?
- How are the distance formula and the Pythagorean theorem the same? Different?
- How are the volume formulas for cones, cylinders, prisms and pyramids interrelated?
- How is volume of an irregular figure measured?
- How can cubic units be used to measure volume for curved surfaces?

###### Relevance and Application:

- The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings.
- Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads.
- The ability to find volume helps to answer important questions such as how to minimize waste by redesigning packaging or maximizing volume by using a circular base.

###### Nature of Mathematics:

- Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden.
- Geometric objects are abstracted and simplified versions of physical objects
- Mathematicians make sense of problems and persevere in solving them. (MP)
- Mathematicians construct viable arguments and critique the reasoning of others. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Eighth Grade**

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<sup>1</sup> Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.1a)

Angles are taken to angles of the same measure. (CCSS: 8.G.1b)

Parallel lines are taken to parallel lines. (CCSS: 8.G.1c)

<sup>2</sup> For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS: 8.G.5)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: Seventh Grade**

**Concepts and skills students master:**

1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof

**Evidence Outcomes**

**Students can:**

- a. Draw construct, and describe geometrical figures and describe the relationships between them. (CCSS: 7.G)
  - i. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.1)
  - ii. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. (CCSS: 7.G.2)
  - iii. Construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.2)
  - iv. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. Is there a geometric figure for any given set of attributes?
- 2. How does scale factor affect length, perimeter, angle measure, area and volume?
- 3. How do you know when a proportional relationship exists?

**Relevance and Application:**

- 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography.
- 2. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures.

**Nature of Mathematics:**

- 1. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning.
- 2. The relationship between geometric figures can be modeled
- 3. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings.
- 4. Mathematicians use appropriate tools strategically. (MP)
- 5. Mathematicians attend to precision. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

#### Grade Level Expectation: Seventh Grade

##### Concepts and skills students master:

2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure

#### Evidence Outcomes

##### Students can:

- a. State the formulas for the area and circumference of a circle and use them to solve problems. (CCSS: 7.G.4)
- b. Give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.4)
- c. Use properties of supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.5)
- d. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.6)

#### 21<sup>st</sup> Century Skills and Readiness Competencies

##### Inquiry Questions:

1. How can geometric relationships among lines and angles be generalized, described, and quantified?
2. How do line relationships affect angle relationships?
3. Can two shapes have the same volume but different surface areas? Why?
4. Can two shapes have the same surface area but different volumes? Why?
5. How are surface area and volume like and unlike each other?
6. What do surface area and volume tell about an object?
7. How are one-, two-, and three-dimensional units of measure related?
8. Why is pi an important number?

##### Relevance and Application:

1. The ability to find volume and surface area helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use.

##### Nature of Mathematics:

1. Geometric objects are abstracted and simplified versions of physical objects.
2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects.
3. Mathematicians make sense of problems and persevere in solving them. (MP)
4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)



**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**  
➤ Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

**Grade Level Expectation: Sixth Grade**

**Concepts and skills students master:**  
1. Objects in space and their parts and attributes can be measured and analyzed

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can</b></p> <p>a. Develop and apply formulas and procedures for area of plane figures</p> <ul style="list-style-type: none"><li>i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. (CCSS: 6.G.1)</li><li>ii. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.1)</li></ul> <p>b. Develop and apply formulas and procedures for volume of regular prisms.</p> <ul style="list-style-type: none"><li>i. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. (CCSS: 6.G.2)</li><li>ii. Show that volume is the same as multiplying the edge lengths of a rectangular prism. (CCSS: 6.G.2)</li><li>iii. Apply the formulas <math>V = lwh</math> and <math>V = bh</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.2)</li></ul> <p>c. Draw polygons in the coordinate plane to solve real-world and mathematical problems. (CCSS: 6.G.3)</p> <ul style="list-style-type: none"><li>i. Draw polygons in the coordinate plane given coordinates for the vertices.</li><li>ii. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. (CCSS: 6.G.3)</li></ul> <p>d. Develop and apply formulas and procedures for the surface area.</p> <ul style="list-style-type: none"><li>i. Represent three-dimensional figures using nets made up of rectangles and triangles. (CCSS: 6.G.4)</li><li>ii. Use nets to find the surface area of figures. (CCSS: 6.G.4)</li><li>iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems. (CCSS: 6.G.4)</li></ul>	<p><b>Inquiry Questions:</b></p> <ul style="list-style-type: none"><li>1. Can two shapes have the same volume but different surface areas? Why?</li><li>2. Can two figures have the same surface area but different volumes? Why?</li><li>3. What does area tell you about a figure?</li><li>4. What properties affect the area of figures?</li></ul> <p><b>Relevance and Application:</b></p> <ul style="list-style-type: none"><li>1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project.</li><li>2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes.</li></ul> <p><b>Nature of Mathematics:</b></p> <ul style="list-style-type: none"><li>1. Mathematicians realize that measurement always involves a certain degree of error.</li><li>2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning.</li><li>3. Mathematicians make sense of problems and persevere in solving them. (MP)</li><li>4. Mathematicians reason abstractly and quantitatively. (MP)</li></ul>

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Fifth Grade**

**Concepts and skills students master:**

1. Properties of multiplication and addition provide the foundation for volume an attribute of solids.

**Evidence Outcomes**

**Students can:**

- a. Model and justify the formula for volume of rectangular prisms. (CCSS: 5.MD.5b)
  - i. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes.<sup>1</sup> (CCSS: 5.MD.5b)
  - ii. Show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. (CCSS: 5.MD.5a)
  - iii. Represent threefold whole-number products as volumes to represent the associative property of multiplication. (CCSS: 5.MD.5a)
- b. Find volume of rectangular prisms using a variety of methods and use these techniques to solve real world and mathematical problems. (CCSS: 5.MD.5a)
  - i. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.4)
  - ii. Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths. (CCSS: 5.MD.5b)
  - iii. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. (CCSS: 5.MD.5c)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. Why do you think a unit cube is used to measure volume?

**Relevance and Application:**

1. The ability to find volume helps to answer important questions such as which container holds more.

**Nature of Mathematics:**

1. Mathematicians create visual and physical representations of problems and ideas that reveal relationships and meaning.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians model with mathematics. (MP)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

**Grade Level Expectation: Fifth Grade**

**Concepts and skills students master:**

2. Geometric figures can be described by their attributes and specific locations in the plane

**Evidence Outcomes**

**Students can:**

- a. Graph points on the coordinate plane<sup>2</sup> to solve real-world and mathematical problems. (CCSS: 5.G)
- b. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.2)
- c. Classify two-dimensional figures into categories based on their properties. (CCSS: 5.G)
  - i. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.<sup>3</sup> (CCSS: 5.G.3)
  - ii. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.4)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How does using a coordinate grid help us solve real world problems?
  - 1. What are the ways to compare and classify geometric figures?
  - 2. Why do we classify shapes?

**Relevance and Application:**

- 1. The coordinate grid is a basic example of a system for mapping relative locations of objects. It provides a basis for understanding latitude and longitude, GPS coordinates, and all kinds of geographic maps.
- 2. Symmetry is used to analyze features of complex systems and to create worlds of art. For example symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper.

**Nature of Mathematics:**

- 1. Geometry's attributes give the mind the right tools to consider the world around us.
- 2. Mathematicians model with mathematics. (MP)
- 3. Mathematicians look for and make use of structure. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Fifth Grade**

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<sup>1</sup> A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. (CCSS: 5.MD.3a)

A solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units. (CCSS: 5.MD.3b)

<sup>2</sup> Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. (CCSS: 5.G.1)

Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,  $x$ -axis and  $x$ -coordinate,  $y$ -axis and  $y$ -coordinate). (CCSS: 5.G.1)

<sup>3</sup> For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. (CCSS: 5.G.3)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time

##### Evidence Outcomes

###### Students can:

- a. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (CCSS: 4.MD)
  - i. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (CCSS: 4.MD.1)
  - ii. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.<sup>1</sup> (CCSS: 4.MD.1)
  - iii. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. (CCSS: 4.MD.2)
  - iv. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.2)
  - v. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.<sup>2</sup> (CCSS: 4.MD.3)
- b. Use concepts of angle and measure angles. (CCSS: 4.MD)
  - i. Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement.<sup>3</sup> (CCSS: 4.MD.5)
  - ii. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.6)
  - iii. Demonstrate that angle measure as additive.<sup>4</sup> (CCSS: 4.MD.7)
  - iv. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.<sup>5</sup> (CCSS: 4.MD.7)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. How do you decide when close is close enough?
2. How can you describe the size of geometric figures?

###### Relevance and Application:

1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture.

###### Nature of Mathematics:

1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade.
2. Mathematicians make sense of problems and persevere in solving them. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: Fourth Grade

##### Concepts and skills students master:

2. Geometric figures in the plane and in space are described and analyzed by their attributes

##### Evidence Outcomes

###### Students can:

- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. (CCSS: 4.G.1)
- Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. (CCSS: 4.G.1)
- Classify and identify two-dimensional figures according to attributes of line relationships or angle size.<sup>6</sup> (CCSS: 4.G.2)
- Identify a line of symmetry for a two-dimensional figure.<sup>7</sup> (CCSS: 4.G.3)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- How do geometric relationships help us solve problems?
- Is a square still a square if it's tilted on its side?
- How are three-dimensional shapes different from two-dimensional shapes?
- What would life be like in a two-dimensional world?
- Why is it helpful to classify things like angles or shapes?

###### Relevance and Application:

- The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned.
- The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam.

###### Nature of Mathematics:

- Geometry is a system that can be used to model the world around us or to model imaginary worlds.
- Mathematicians look for and make use of structure. (MP)
- Mathematicians look for and express regularity in repeated reasoning. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Fourth Grade**

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<sup>1</sup> For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS: 4.MD.1)

<sup>2</sup> For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS: 4.MD.3)

<sup>3</sup> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.5a)

An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees. (CCSS: 4.MD.5b)

<sup>4</sup> When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. (CCSS: 4.MD.7)

<sup>5</sup> e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.7)

<sup>6</sup> Based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.2)

<sup>7</sup> as a line across the figure such that the figure can be folded along the line into matching parts. (CCSS: 4.G.3)

Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.3)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

1. Geometric figures are described by their attributes

**Evidence Outcomes**

**Students can:**

- a. Reason with shapes and their attributes. (CCSS: 3.G)
  - i. Explain that shapes in different categories<sup>1</sup> may share attributes<sup>2</sup> and that the shared attributes can define a larger category.<sup>3</sup> (CCSS: 3.G.1)
    1. Identify rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.1)
  - ii. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.<sup>4</sup> (CCSS: 3.G.2)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. What words in geometry are also used in daily life?
2. Why can different geometric terms be used to name the same shape?

**Relevance and Application:**

1. Recognition of geometric shapes allows people to describe and change their surroundings such as creating a work of art using geometric shapes, or design a pattern to decorate.

**Nature of Mathematics:**

1. Mathematicians use clear definitions in discussions with others and in their own reasoning.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians look for and make use of structure. (MP)



**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

- 2. Linear and area measurement are fundamentally different and require different units of measure

**Evidence Outcomes**

**Students can:**

- a. Use concepts of area and relate area to multiplication and to addition. (CCSS: 3.MD)
  - i. Recognize area as an attribute of plane figures and apply concepts of area measurement.<sup>5</sup> (CCSS: 3.MD.5)
  - ii. Find area of rectangles with whole number side lengths using a variety of methods<sup>6</sup> (CCSS: 3.MD.7a)
  - iii. Relate area to the operations of multiplication and addition and recognize area as additive.<sup>7</sup> (CCSS: 3.MD.7)
- b. Describe perimeter as an attribute of plane figures and distinguish between linear and area measures. (CCSS: 3.MD)
- c. Solve real world and mathematical problems involving perimeters of polygons. (CCSS: 3.MD.8)
  - i. Find the perimeter given the side lengths. (CCSS: 3.MD.8)
  - ii. Find an unknown side length given the perimeter. (CCSS: 3.MD.8)
  - iii. Find rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.8)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. What kinds of questions can be answered by measuring?
- 2. What are the ways to describe the size of an object or shape?
- 3. How does what we measure influence how we measure?
- 4. What would the world be like without a common system of measurement?

**Relevance and Application:**

- 1. The use of measurement tools allows people to gather, organize, and share data with others such as sharing results from science experiments, or showing the growth rates of different types of seeds.
- 2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods.

**Nature of Mathematics:**

- 1. Mathematicians use tools and techniques to accurately determine measurement.
- 2. People use measurement systems to specify attributes of objects with enough precision to allow collaboration in production and trade.
- 3. Mathematicians make sense of problems and persevere in solving them. (MP)
- 4. Mathematicians model with mathematics. (MP)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Third Grade**

**Concepts and skills students master:**

3. Time and attributes of objects can be measured with appropriate tools

**Evidence Outcomes**

**Students can:**

- a. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (CCSS: 3.MD)
  - i. Tell and write time to the nearest minute. (CCSS: 3.MD.1)
  - ii. Measure time intervals in minutes. (CCSS: 3.MD.1)
  - iii. Solve word problems involving addition and subtraction of time intervals in minutes<sup>8</sup> using a number line diagram. (CCSS: 3.MD.1)
  - iv. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (CCSS: 3.MD.2)
  - v. Use models to add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units.<sup>9</sup> (CCSS: 3.MD.2)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. Why do we need standard units of measure?
- 2. Why do we measure time?

**Relevance and Application:**

- 1. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods.

**Nature of Mathematics:**

- 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade.
- 2. Mathematicians use appropriate tools strategically. (MP)
- 3. Mathematicians attend to precision. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Third Grade**

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<sup>1</sup> e.g., rhombuses, rectangles, and others. (CCSS: 3.G.1)

<sup>2</sup> e.g., having four sides. (CCSS: 3.G.1)

<sup>3</sup> e.g., quadrilaterals. (CCSS: 3.G.1)

<sup>4</sup> For example, partition a shape into 4 parts with equal area, and describe the area of each part as  $\frac{1}{4}$  of the area of the shape. (CCSS: 3.G.2)

<sup>5</sup> A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)

A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units. (CCSS: 3.MD.5b)

<sup>6</sup> A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.5a)

A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units. (CCSS: 3.MD.5b)

Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.6)

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.7a)

Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.7b)

<sup>7</sup> Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (CCSS: 3.MD.7d)

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ .

Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.7c)

<sup>8</sup> e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.1)

<sup>9</sup> e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (CCSS: 3.MD.2)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Apply transformation to numbers, shapes, functional representations, and data

**Grade Level Expectation: Second Grade**

**Concepts and skills students master:**

1. Shapes can be described by their attributes and used to represent part/whole relationships

**Evidence Outcomes**

**Students can:**

- a. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (CCSS: 2.G.1)
- b. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.1)
- c. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.2)
- d. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. (CCSS: 2.G.3)
- e. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. How can we describe geometric figures?
2. Is a half always the same size and shape?

**Relevance and Application:**

1. Fairness in sharing depends on equal quantities, such as sharing a piece of cake, candy bar, or payment for a chore.
2. Shapes are used to communicate how people view their environment.
3. Geometry provides a system to describe, organize, and represent the world around us.

**Nature of Mathematics:**

1. Geometers use shapes to describe and understand the world.
2. Mathematicians reason abstractly and quantitatively. (MP)
3. Mathematicians model with mathematics. (MP)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

#### Grade Level Expectation: Second Grade

##### Concepts and skills students master:

2. Some attributes of objects are measurable and can be quantified using different tools

##### Evidence Outcomes

###### Students can:

- Measure and estimate lengths in standard units. (CCSS: 2.MD)
  - Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.1)
  - Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.2)
  - Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.3)
  - Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.4)
- Relate addition and subtraction to length. (CCSS: 2.MD)
  - Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units<sup>1</sup> and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.5)
  - Represent whole numbers as lengths from 0 on a number line<sup>2</sup> diagram and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.6)
- Solve problems time and money. (CCSS: 2.MD)
  - Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.7)
  - Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately.<sup>3</sup> (CCSS: 2.MD.8)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

- What are the different things we can measure?
- How do we decide which tool to use to measure something?
- What would happen if everyone created and used their own rulers?

###### Relevance and Application:

- Measurement is used to understand and describe the world including sports, construction, and explaining the environment.

###### Nature of Mathematics:

- Mathematicians use measurable attributes to describe countless objects with only a few words.
- Mathematicians use appropriate tools strategically. (MP)
- Mathematicians attend to precision. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**Second Grade**

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<sup>1</sup> e.g., by using drawings (such as drawings of rulers). (CCSS: 2.MD.5)

<sup>2</sup> with equally spaced points corresponding to the numbers 0, 1, 2, ... (CCSS: 2.MD.6)

<sup>3</sup> Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS: 2.MD.6)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

**Grade Level Expectation: First Grade**

**Concepts and skills students master:**

1. Shapes can be described by defining attributes and created by composing and decomposing

<b>Evidence Outcomes</b>	<b>21<sup>st</sup> Century Skills and Readiness Competencies</b>
<p><b>Students can:</b></p> <ul style="list-style-type: none"><li>a. Distinguish between defining attributes<sup>1</sup> versus non-defining attributes.<sup>2</sup> (CCSS: 1.G.1)</li><li>b. Build and draw shapes to possess defining attributes. (CCSS: 1.G.1)</li><li>c. Compose two-dimensional shapes<sup>3</sup> or three-dimensional shapes<sup>4</sup> to create a composite shape, and compose new shapes from the composite shape. (CCSS: 1.G.2)</li><li>d. Partition circles and rectangles into two and four equal shares. (CCSS: 1.G.3)<ul style="list-style-type: none"><li>i. Describe shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. (CCSS: 1.G.3)</li><li>ii. Describe the whole as two of, or four of the equal shares.<sup>5</sup> (CCSS: 1.G.3)</li></ul></li></ul>	<p><b>Inquiry Questions:</b></p> <ul style="list-style-type: none"><li>1. What shapes can be combined to create a square?</li><li>2. What shapes can be combined to create a circle?</li></ul> <p><b>Relevance and Application:</b></p> <ul style="list-style-type: none"><li>1. Many objects in the world can be described using geometric shapes and relationships such as architecture, objects in your home, and things in the natural world. Geometry gives us the language to describe these objects.</li><li>2. Representation of ideas through drawing is an important form of communication. Some ideas are easier to communicate through pictures than through words such as the idea of a circle, or an idea for the design of a couch.</li></ul> <p><b>Nature of Mathematics:</b></p> <ul style="list-style-type: none"><li>1. Geometers use shapes to represent the similarity and difference of objects.</li><li>2. Mathematicians model with mathematics. (MP)</li><li>3. Mathematicians look for and make use of structure. (MP)</li></ul>

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: First Grade**

**Concepts and skills students master:**

2. Measurement is used to compare and order objects and events

**Evidence Outcomes**

**Students can:**

- a. Measure lengths indirectly and by iterating length units. (CCSS: 1.MD)
  - i. Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS: 1.MD.1)
  - ii. Express the length of an object as a whole number of length units.<sup>6</sup> (CCSS: 1.MD.2)
- b. Tell and write time. (CCSS: 1.MD)
  - i. Tell and write time in hours and half-hours using analog and digital clocks. (CCSS: 1.MD.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How can you tell when one thing is bigger than another?
- 2. Why do we measure objects and time?
- 3. How are length and time different? How are they the same?

**Relevance and Application:**

- 1. Time measurement is a means to organize and structure each day and our lives, and to describe tempo in music.
- 2. Measurement helps to understand and describe the world such as comparing heights of friends, describing how heavy something is, or how much something holds.

**Nature of Mathematics:**

- 1. With only a few words, mathematicians use measurable attributes to describe countless objects.
- 2. Mathematicians use appropriate tools strategically. (MP)
- 3. Mathematicians attend to precision. (MP)



**Standard: 4. Shape, Dimension, and Geometric Relationships**  
**First Grade**

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<sup>1</sup> e.g., triangles are closed and three-sided. (CCSS: 1.G.1)

<sup>2</sup> e.g., color, orientation, overall size. (CCSS: 1.G.1)

<sup>3</sup> rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles. (CCSS: 1.G.2)

<sup>4</sup> cubes, right rectangular prisms, right circular cones, and right circular cylinders. (CCSS: 1.G.2)

<sup>5</sup> Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS: 1.G.3)

<sup>6</sup> By laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. (CCSS: 1.MD.2)

## Content Area: Mathematics

### Standard: 4. Shape, Dimension, and Geometric Relationships

#### Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

#### Grade Level Expectation: Kindergarten

##### Concepts and skills students master:

1. Shapes can be described by characteristics and position and created by composing and decomposing

##### Evidence Outcomes

Students can:

- a. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). (CCSS: K.G)
  - i. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.1)
  - ii. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.2)
  - iii. Identify shapes as two-dimensional<sup>1</sup> or three dimensional.<sup>2</sup> (CCSS: K.G.3)
- b. Analyze, compare, create, and compose shapes. (CCSS: K.G)
  - i. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts<sup>3</sup> and other attributes.<sup>4</sup> (CCSS: K.G.4)
  - ii. Model shapes in the world by building shapes from components<sup>5</sup> and drawing shapes. (CCSS: K.G.5)
  - iii. Compose simple shapes to form larger shapes.<sup>6</sup> (CCSS: K.G.6)

##### 21<sup>st</sup> Century Skills and Readiness Competencies

###### Inquiry Questions:

1. What are the ways to describe where an object is?
2. What are all the things you can think of that are round? What is the same about these things?
3. How are these shapes alike and how are they different?
4. Can you make one shape with other shapes?

###### Relevance and Application:

1. Shapes help people describe the world. For example, a box is a cube, the Sun looks like a circle, and the side of a dresser looks like a rectangle.
2. People communicate where things are by their location in space using words like next to, below, or between.

###### Nature of Mathematics:

1. Geometry helps discriminate one characteristic from another.
2. Geometry clarifies relationships between and among different objects.
3. Mathematicians model with mathematics. (MP)
4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Kindergarten**

**Concepts and skills students master:**

**2. Measurement is used to compare and order objects**

**Evidence Outcomes**

**Students can:**

- a. Describe and compare measurable attributes. (CCSS: K.MD)
  - i. Describe measurable attributes of objects, such as length or weight. (CCSS: K.MD.1)
  - ii. Describe several measurable attributes of a single object. (CCSS: K.MD.1)
  - iii. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference.<sup>7</sup> (CCSS: K.MD.2)
  - iv. Order several objects by length, height, weight, or price (PFL)
- b. Classify objects and count the number of objects in each category. (CCSS: K.MD)
  - i. Classify objects into given categories. (CCSS: K.MD.3)
  - ii. Count the numbers of objects in each category. (CCSS: K.MD.3)
  - iii. Sort the categories by count. (CCSS: K.MD.3)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How can you tell when one thing is bigger than another?
- 2. How is height different from length?

**Relevance and Application:**

- 1. Measurement helps to understand and describe the world such as in cooking, playing, or pretending.
- 2. People compare objects to communicate and collaborate with others. For example, we describe items like the long ski, the heavy book, the expensive toy.

**Nature of Mathematics:**

- 1. A system of measurement provides a common language that everyone can use to communicate about objects.
- 2. Mathematicians use appropriate tools strategically. (MP)
- 3. Mathematicians attend to precision. (MP)

**Standard: 4. Shape, Dimension, and Geometric Relationships  
Kindergarten**

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<sup>1</sup> lying in a plane, “flat”. (CCSS: K.G.3)

<sup>2</sup> “solid”. (CCSS: K.G.3)

<sup>3</sup> e.g., number of sides and vertices/“corners”. (CCSS: K.G.4)

<sup>4</sup> e.g., having sides of equal length. (CCSS: K.G.4)

<sup>5</sup> e.g., sticks and clay balls. (CCSS: K.G.5)

<sup>6</sup> For example, “Can you join these two triangles with full sides touching to make a rectangle?” (CCSS: K.G.6)

<sup>7</sup> For example, directly compare the heights of two children and describe one child as taller/shorter. (CCSS: K.MD.2)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

**Grade Level Expectation: Preschool**

**Concepts and skills students master:**

1. Shapes can be observed in the world and described in relation to one another

**Evidence Outcomes**

**Students can:**

- a. Match, sort, group and name basic shapes found in the natural environment
- b. Sort similar groups of objects into simple categories based on attributes
- c. Use words to describe attributes of objects
- d. Follow directions to arrange, order, or position objects

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

1. How do we describe where something is?
2. Where do you see shapes around you?
3. How can we arrange these shapes?
4. Why do we put things in a group?
5. What is the same about these objects and what is different?
6. What are the ways to sort objects?

**Relevance and Application:**

1. Shapes and position help students describe and understand the environment such as in cleaning up, or organizing and arranging their space.
2. Comprehension of order and position helps students learn to follow directions.
3. Technology games can be used to arrange and position objects.
4. Sorting and grouping allows people to organize their world. For example, we set up time for clean up, and play.

**Nature of Mathematics:**

1. Geometry affords the predisposition to explore and experiment.
2. Mathematicians organize objects in different ways to learn about the objects and a group of objects.
3. Mathematicians attend to precision. (MP)
4. Mathematicians look for and make use of structure. (MP)

**Content Area: Mathematics**

**Standard: 4. Shape, Dimension, and Geometric Relationships**

**Prepared Graduates:**

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Grade Level Expectation: Preschool**

**Concepts and skills students master:**

2. Measurement is used to compare objects

**Evidence Outcomes**

**Students can:**

- a. Describe the order of common events
- b. Group objects according to their size using standard and non-standard forms (height, weight, length, or color brightness) of measurement
- c. Sort coins by physical attributes such as color or size (PFL)

**21<sup>st</sup> Century Skills and Readiness Competencies**

**Inquiry Questions:**

- 1. How do we know how big something is?
- 2. How do we describe when things happened?

**Applying Mathematics in Society and Using Technology:**

- 1. Understanding the order of events allows people to tell a story or communicate about the events of the day.
- 2. Measurements helps people communicate about the world. For example, we describe items like big and small cars, short and long lines, or heavy and light boxes.

**Nature of Mathematics:**

- 1. Mathematicians sort and organize to create patterns. Mathematicians look for patterns and regularity. The search for patterns can produce rewarding shortcuts and mathematical insights.
- 2. Mathematicians reason abstractly and quantitatively. (MP)
- 3. Mathematicians use appropriate tools strategically. (MP)

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